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A General Model of Fuzzy Plasticity

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Abstract: The transition between the elastic and plastic states is sharp in the classical plasticity theory. To overcome this problem, many constitutive models, such as multi-yield-surface model and two-surface model, have been developed. However, these models can not represent the true deformation process in a material. In order to capture nonlinear hardening behavior and smooth transition from elastic to plastic state, a general model of fuzzy plasticity is developed. On the basis of the theory of fuzzy sets and TAKAGI-SUGENO fuzzy model, a fuzzy plastic model for monotonic and cyclic loadings in one dimension is established and it is generalized to six dimensions and unsymmetric cycles. The proposed model uses a set of surfaces to partition the stress space with individual plastic modulus. The plastic modulus between two adjacent surfaces is determined by a membership function. By means of a finite number of partitioning surfaces, the fuzzy plastic model can provide with a more realistic and practical description of the materials behavior than the classical plasticity model. The validity of the fuzzy plastic model is investigated by comparing the predicted and experimental stress-strain responses of steels. It was found that the fuzzy plasticity has the ability to handle many practical problems that cannot be adequately analyzed by the conventional theory of plasticity.

Key words: plasticity theory, fuzzy model, membership function, plastic modulus, stress-strain curve

1 Introduction

Constitutive modelling of plasticity has been a research focus over many decades. For example, MROZ introduced the concept of a “field of work hardening moduli” and generalized the linear kinematic hardening model[1–3], which led to multi-yield-surface model [4–6], approximated by a multilinear response of a material. For each linear part, a linear kinematic hardening model is used with a specific value of plastic modulus. One of the main difficulties with the multi-yield-surface model is the large number of surfaces necessary to describe material’s hardening, and each surface needs the storage of a tensor variable (usually six components) and a scalar variable. To overcome such problem, further development has been trying to obtain the same specific properties by using only two surfaces, i.e., the yield surface and the bounding surface or limit surface[7], such as bounding surface plasticity[8–11], subloadingsurface elastoplasticity[12], and discrete memory[13,14]. Instead of a constant modulus, these two-surface models use a plastic modulus that varies in a continuous manner between the two surfaces. However, HASHIGUCHI[15] pointed out that bounding surface plasticity produces excessively large ratchetting. In an attempt to mitigate this artificial ratcheting, BARDET [16–18] proposed a constitutive model based on the concept of scaled memory. GRANLUND[19] and OLSSON[20] developed a two-surface concept, using an elastic limit surface and a memory surface, to describe the behaviour of structural steel in non-monotonic loading situation. The transition from elastic to plastic state in subsequent loading is described by fuzzy-set plasticity of KLISINSKI[21].

In order to capture the nonlinear hardening behavior, a somewhat different approach was introduced by KLISINSKI, et al[21,22], based on the theory of fuzzy sets, which assumed that there exists an ultimate yield surface where the behavior of the material is entirely plastic. Within this ultimate surface both elastic and plastic strains develop on the current yield surface, and the response of the material inside the initial yield surface is purely elastic. But the elastoplastic response between the initial and the ultimate surfaces is characterized by a fuzzy set. Every point in this region is assigned with a real number $\gamma(\boldsymbol{\sigma})$ on the interval [0,1]. If the stress point is on the ultimate yield surface, the membership function γ is zero, but for purely elastic behavior γ become 1. Scalar $\gamma(\boldsymbol{\sigma})$ represents the degree of membership that stress $\boldsymbol{\sigma}$ has to be the set of stresses exhibiting purely elastic behavior. KLISINSKI, et al, used a new plastic modulus h to replace the plastic modulus H in the conventional formulation of hardening incremental plasticity. This new plastic modulus h is a

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function of the plastic modulus H and the value of the membership function

$$h = h[H, \gamma] \qquad (1)$$

This function should satisfy the following conditions

$$\begin{aligned} h(H,1) &= \infty \\ h(H,0) &= H \end{aligned}, \qquad (2)$$

An example of the form of this function is

$$h(H,\gamma) = H + (M - H)\gamma^a; \qquad a \neq 0 . \qquad (3)$$

where M is a large positive number.

In the above fuzzy plasticity theory, there is only one membership function $\gamma(\boldsymbol{\sigma})$. Its key is to find the new plastic modulus h. The nonlinear function h is defined in terms of the value of membership function $\gamma(\boldsymbol{\sigma})$. Similarly, to determine the plastic modulus H in the traditional way, it is difficult to determine function $\gamma(\boldsymbol{\sigma})$. As a matter of fact, it is not a real fuzzy model.

The present paper aims to develop a general fuzzy plastic model to describe the behavior of materials. On the basis of the theory of fuzzy sets[23] and TAKAGI-SUGENO fuzzy model[24], we first establish a fuzzy plastic model for monotonic and cyclic loadings in one dimension, we will then generalize it to six dimensions and unsymmetric cycles. The new model uses a set of surfaces to partition the stress space, with each surface having a constant plastic modulus. The plastic modulus between two adjacent surfaces is determined by a membership function. By means of a finite number of partitioning surfaces, the model can provide with a more realistic and practical description of stress-strain curves.

2 One-Dimensional Fuzzy Plastic Model

2.1 Classical plasticity theory

First let us briefly review the classical plasticity theory. As we know, the conventional incremental theory of plasticity is based on the assumption that there exists a yield surface in the stress space

$$\boldsymbol{f}(\boldsymbol{\sigma}, \boldsymbol{\kappa}) = 0 , \qquad (4)$$

where $\boldsymbol{\kappa}$ is a hardening parameter. The yield surface defines the elastic region of a material. When a stress point is within the yield surface (i.e., $\boldsymbol{f}(\boldsymbol{\sigma}, \boldsymbol{\kappa}) < 0$), only elastic strains take place and the increment of plastic strain is zero. When a stress point is on the yield surface (i.e., $\boldsymbol{f}(\boldsymbol{\sigma}, \boldsymbol{\kappa}) = 0$), plastic strains occur. The basic formulation of the constitutive relations is

$$\mathrm{d}\boldsymbol{\varepsilon} = \mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{e}} + \mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}} , \qquad (5)$$

where

$$\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{e}} = \boldsymbol{C}^{\mathrm{e}} \mathrm{d}\boldsymbol{\sigma} , \qquad (6)$$

$$\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}} = \mathrm{d}\zeta \boldsymbol{m}; \quad \mathrm{d}\zeta \geq 0 . \qquad (7)$$

In above expressions, $\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{e}}$ is the elastic strain increment, $\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}$ the plastic strain increment, $\boldsymbol{C}^{\mathrm{e}}$ the elastic compliance matrix, $\mathrm{d}\zeta$ a plastic multiplier, $\boldsymbol{m} = \partial \boldsymbol{g}(\boldsymbol{\sigma}) / \partial \boldsymbol{\sigma}$ the flow direction, and $\boldsymbol{g}(\boldsymbol{\sigma})$ the plastic potential function.
When $\boldsymbol{f}(\boldsymbol{\sigma}, \boldsymbol{\kappa}) = 0$, the loading / unloading criteria are

$$\begin{aligned} &\mathrm{d}\zeta > 0; \mathrm{d}\boldsymbol{f} = 0, \text{ plastic loading} \\ &\mathrm{d}\zeta = 0; \mathrm{d}\boldsymbol{f} = 0, \text{ neutral loading} \\ &\mathrm{d}\zeta = 0; \mathrm{d}\boldsymbol{f} < 0, \text{ elastic unloading} \end{aligned} , \qquad (8)$$

The increments of plastic strain occur only in the loading case, the plastic multiplier can be obtained

$$\mathrm{d}\zeta = \frac{1}{H} \boldsymbol{n} \mathrm{d}\boldsymbol{\sigma} , \qquad (9)$$

where $\boldsymbol{n} = \partial \boldsymbol{f} / \partial \boldsymbol{\sigma}$ is the gradient of the yield surface.

The plastic strain increment can be written in the following form

$$\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}} = \begin{cases} \dfrac{1}{H} (\boldsymbol{n} \mathrm{d}\boldsymbol{\sigma}) \boldsymbol{m}; & \text{if } \boldsymbol{f} = 0; \ \boldsymbol{n} \mathrm{d}\boldsymbol{\sigma} \geq 0 \\ 0; \ \text{if } \boldsymbol{f} < 0 \text{ or } \boldsymbol{f} = 0; & \boldsymbol{n} \mathrm{d}\boldsymbol{\sigma} \leq 0 \end{cases} , \qquad (10)$$

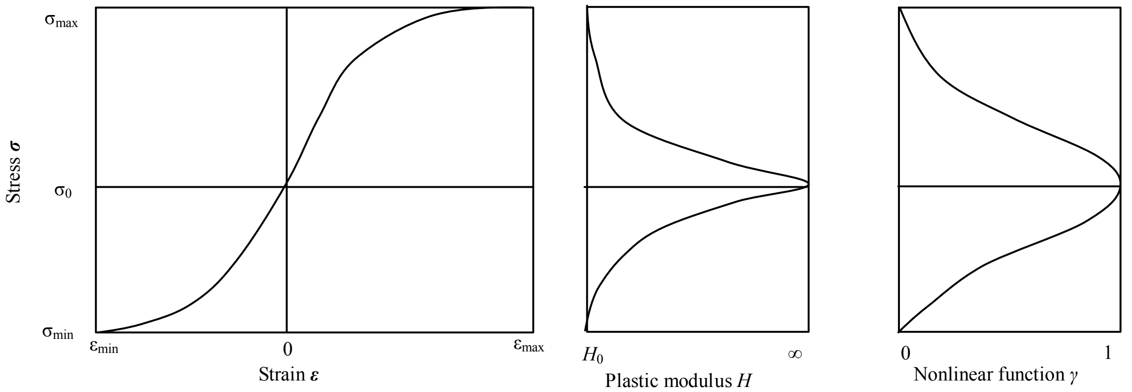
For a work hardening material, the strain increment is normal to the loading surface and hence, $\boldsymbol{g} = \boldsymbol{f}$. This means that the plastic potential and the yield surface are the same and we have $\boldsymbol{n} = \boldsymbol{m}$, resulting in the associated flow rule[25, 26, 21]. The transition between the elastic and plastic states is sharp in this theory, i.e., there is not a transition zone between elasticity and plasticity, which does not represent the true deformation process in a material [27].

2.2 Fuzzy plastic model for monotonic loadings

In an attempt to improve the description of the behavior of materials, we propose a general concept of fuzzy plastic model based on the theory of fuzzy sets and TAKAGI-SUGENO fuzzy model. Here, we will first define the fuzzy plastic model in one-dimensional monotonic loadings.

As shown in Fig. 1, the initial state is strain- and stress-free (i.e., $\boldsymbol{\varepsilon} = 0$, and $\boldsymbol{\sigma} = \sigma_0 = 0$). $\sigma_{\max}$ and $\sigma_{\min}$ are the compressive and tensile yield stresses, respectively. Similar to elastoplasticity[28], the nonlinear stress-strain response is incrementally described by splitting the incremental strain

$d\varepsilon$ into the elastic strain $d\varepsilon^e$ and plastic strain $d\varepsilon^p$



(a) Stress-strain curve during compression and tension (b) Plastic modulus during compression and tension (c) Nonlinear function γ during compression and tension

Fig.1. Fuzzy plastic model for monotonic loadings

$$d\varepsilon = d\varepsilon^e + d\varepsilon^p = \frac{d\boldsymbol{\sigma}}{E} + \frac{d\boldsymbol{\sigma}}{H}, \tag{11}$$

where $d\boldsymbol{\sigma}$ is stress increment; E is elastic modulus; and H is plastic modulus.

The plastic modulus H can be selected as the following function:

$$H = \frac{H_0}{1-\gamma(\boldsymbol{\sigma})}, \tag{12}$$

where H_0 is the plastic modulus in perfectly plastic state, and $\gamma(\boldsymbol{\sigma})(0\le\gamma\le 1)$ is a nonlinear function of the stress $\boldsymbol{\sigma}$.

At the initial stress state (i.e., $\boldsymbol{\sigma}=0$), $\gamma=1$, $H\to\infty$, Eq.(11) predicts a purely elastic response. When the stress reaches σ_{max} or σ_{min}, $\gamma=0$, H=H_0, Eq. (11) predicts a perfectly plastic response. Eq.(12) defines a gradual transition from purely elastic (i.e., $H\to\infty$) to perfectly plastic responses (i.e., $H=H_0$). Eq. (12) transforms the nonlinear variation of H on the interval $[H_0, \infty]$ shown in Fig.1(b) into the nonlinear variation of γ on the interval [0,1] shown in Fig.1(c).

To characterize this nonlinear function $\gamma(\boldsymbol{\sigma})$ better, we use TAKAGI-SUGENO fuzzy model, which can uniformly approximate a continuous function on a closed interval accurately[29].

2.2.1 *Fuzzification of stress $\boldsymbol{\sigma}$*

N fuzzy sets are used. Each fuzzy set, denoted by A_j (j=1, 2, … , N), has a continuous membership function μ_j . We partition stress space $[\sigma_0, \sigma_{max}]$ into (N-1) intervals, as shown in Fig.2.

2.2.2 *Fuzzy rules*

Rule 1: IF $\boldsymbol{\sigma}$ is A_1 THEN $d\varepsilon^p = \frac{1-\gamma_1}{H_0} d\boldsymbol{\sigma}$ with the membership value w_1

Rule 2: IF $\boldsymbol{\sigma}$ is A_2 THEN $d\varepsilon^p = \frac{1-\gamma_2}{H_0} d\boldsymbol{\sigma}$ with the membership value w_2

⋮

Rule j: IF $\boldsymbol{\sigma}$ is A_j THEN $d\varepsilon^p = \frac{1-\gamma_j}{H_0} d\boldsymbol{\sigma}$ with the membership value w_j

⋮

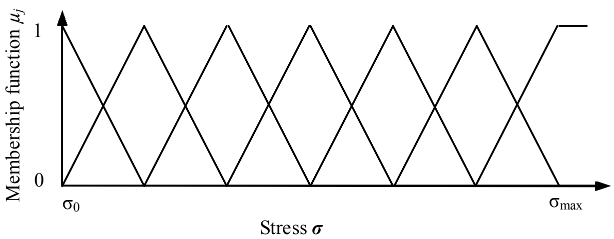
Rule N: IF $\boldsymbol{\sigma}$ is A_N THEN $d\varepsilon^p = \frac{1-\gamma_N}{H_0} d\boldsymbol{\sigma}$ with the membership value w_N

where $\gamma_j\,(0\le\gamma_j\le 1)$ are design parameters.

2.2.2 *Defuzzification*

The centroid defuzzifier is used.

$$d\varepsilon^p = \frac{\sum_{j=1}^{N} w_j \left(\frac{1-\gamma_j}{H_0} d\boldsymbol{\sigma}\right)}{\sum_{j=1}^{N} w_j}, \tag{13}$$



The stress-strain response during one-dimensional monotonic loadings can be calculated by using Eq. (11) and Eq. (13).

Fig. 2. Membership function

2.3 Fuzzy plastic model for cyclic loadings (symmetric stress cycles)

2.3.1 Unloading

During the unloading (i.e. $d\boldsymbol{\sigma}<0$) from point A shown in Fig. 3(a), γ is constructed to obtain the elastic response (i.e., $H\rightarrow\infty$, and $\gamma=1$) at the beginning of the unloading. As shown in Fig.3 (b), point B' on the tensile branch has the same value of γ and H as point A' on the compressive branch during loading. The interval partitioning should be conducted on the stress space $[\sigma_A, -\sigma_A]$, as shown in Fig. 4. We have

At point A: $\boldsymbol{\sigma}=\sigma_A$, $\gamma=1$, $H\rightarrow\infty$;

At point B': $\boldsymbol{\sigma}=-\sigma_A$, $\gamma=\gamma_{B'}=\gamma_A$, $H=H_{B'}$;

where γ_A is the value of γ in point A during loading.

$$\gamma_{B'}=\gamma_A=\frac{\sum_{j=1}^{N} w_j\gamma_j}{\sum_{j=1}^{N} w_j}\ , \tag{14}$$

$$H_{B'}=\frac{H_0}{1-\gamma_A}\ . \tag{15}$$

Similar to monotonic loadings, we can get

$$d\boldsymbol{\varepsilon}^{p}=\frac{\sum_{j=1}^{N} w'_j\left(\frac{1-\gamma'_j}{H_0}d\boldsymbol{\sigma}\right)}{\sum_{j=1}^{N} w'_j}\ . \tag{16}$$

2.3.2 Reloading

During the reloading (i.e. $d\boldsymbol{\sigma}>0$) from point B shown in Fig. 3(a), γ is constructed to obtain the elastic response (i.e., $H\rightarrow\infty$, and $\gamma=1$) at the beginning of the reloading (Fig. 3(c)). As shown in Fig. 3(c), point A'' on the compressive branch has the same value of γ and H as point B'' on the tensile branch during unloading. The interval partitioning should be conducted on the stress space $[\sigma_B, \sigma_A]$. We have:

At point B: $\boldsymbol{\sigma}=\sigma_B$, $\gamma=1$, $H\rightarrow\infty$;

At point A'': $\boldsymbol{\sigma}=\sigma_A$, $\gamma=\gamma_{A''}=\gamma_{B''}$;

where $\gamma_{B''}$ is the value of γ in point B during unloading.

$$\gamma_{A''}=\gamma_{B''}=\frac{\sum_{j=1}^{N} w'_j\gamma'_j}{\sum_{j=1}^{N} w'_j}\ . \tag{17}$$

Similar to monotonic loadings, we can get

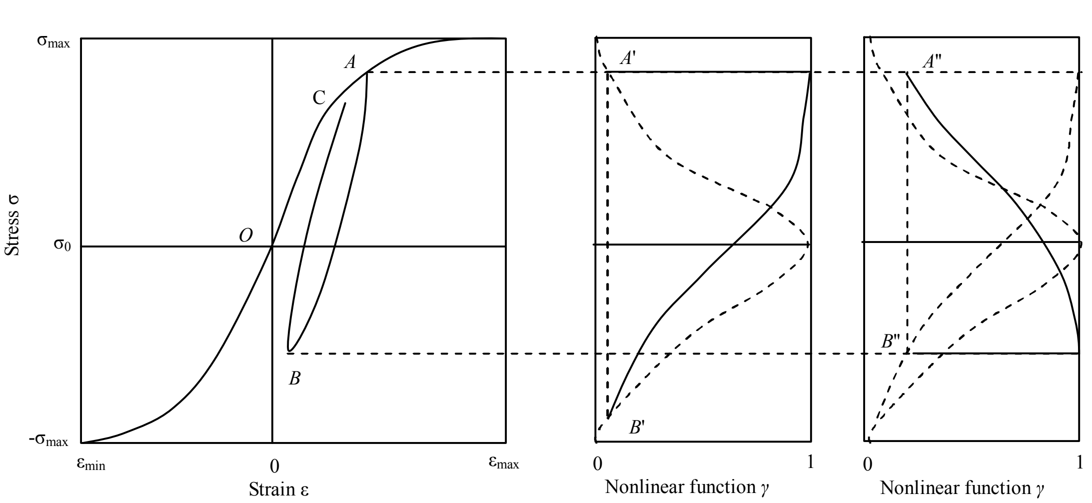
$$d\boldsymbol{\varepsilon}^{p}=\frac{\sum_{j=1}^{N} w''_j\left(\frac{1-\gamma''_j}{H_0}d\boldsymbol{\sigma}\right)}{\sum_{j=1}^{N} w''_j}\ . \tag{18}$$

3 Description of Models with Purely Elastic Region

The purely elastic region may exist from the beginning of the development of plasticity. It can be related to the elastic region enclosed by the first yield surface in the “mechanical sublayer model” advocated by BESSELING[30]. In the bounding surface theory, it is called the elastic nucleus. The value of the nonlinear function γ for points from the purely elastic region is equal to one. The plastic modulus H is infinite.

3.1 Monotonic loadings

As shown in Fig. 5 (a) and (b), at the stress space $[\sigma_0, \sigma_E]$ or $[\sigma_0, -\sigma_E]$, $\gamma=1$, $H\rightarrow\infty$, Eq. (11) predicts a purely elastic response. At the stresses $\sigma_{\max}$ or $\sigma_{\min}$, $\gamma=0$, $H=H_0$, Eq. (11) predicts a perfectly plastic response. The interval partitioning should be conducted on the stress space $[\sigma_E, \sigma_{\max}]$. An equation similar to Eq. (13) can be obtained.



(a) Stress-strain curve during compression, tension, and cyclic loading

(c) Nonlinear function γ during unloading

(c) Nonlinear function γ during reloading

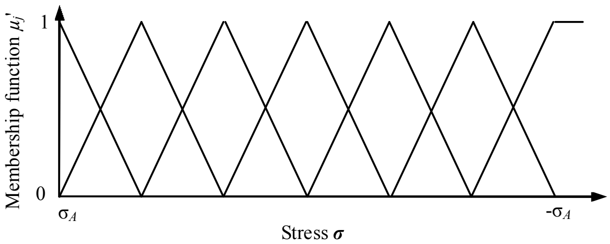


Fig.3. Fuzzy plastic model for cyclic loadings

Fig. 4. Membership Function

3.2 Cyclic Loadings

3.2.1 Unloading

During the unloading from point A shown in Fig. 5(a), γ should represent the elastic response of a material (i.e., $H\rightarrow\infty$, and $\gamma=1$) at the stress space [σ_A, σ_{E1}]. As shown in Fig. 5 (c), point B' on the tensile curve has the same value of γ and H as that at point A' on the compressive curve during loading (i.e., $\boldsymbol{\sigma}=-\sigma_A$, $\gamma=\gamma_A$). To fuzzify stress $\boldsymbol{\sigma}$ on the interval [σ_{E1}, $-\sigma_A$]. An equation similar to Eq. (16) can be obtained.

3.2.2 Reloading

During the reloading from point B shown in Fig. 5(a), γ should represent the elastic response of a material (i.e., $H\rightarrow\infty$, and $\gamma=1$) at the stress space [σ_B, σ_{E2}]. As shown in Fig. 5(d), point A'' on the compressive curve has the same value of γ and H as that at point B'' on the tensile curve during unloading. To fuzzify stress $\boldsymbol{\sigma}$ on the interval [σ_{E2}, σ_A]. An equation similar to Eq. (18) can be obtained.

4. Six-Dimensional Fuzzy Plastic Model

The fuzzy plastic model for one-dimensional loading established as above can be extended to six dimensions.

As in the conventional elastoplasticity[28], the incremental strain is the sum of the elastic strain increment and the plastic strain increment

$$\mathrm{d}\boldsymbol{\varepsilon}=\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{e}}+\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}} \quad , \tag{19}$$

where $\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{e}}$ is the elastic strain increment, $\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}$ is the plastic strain increment.

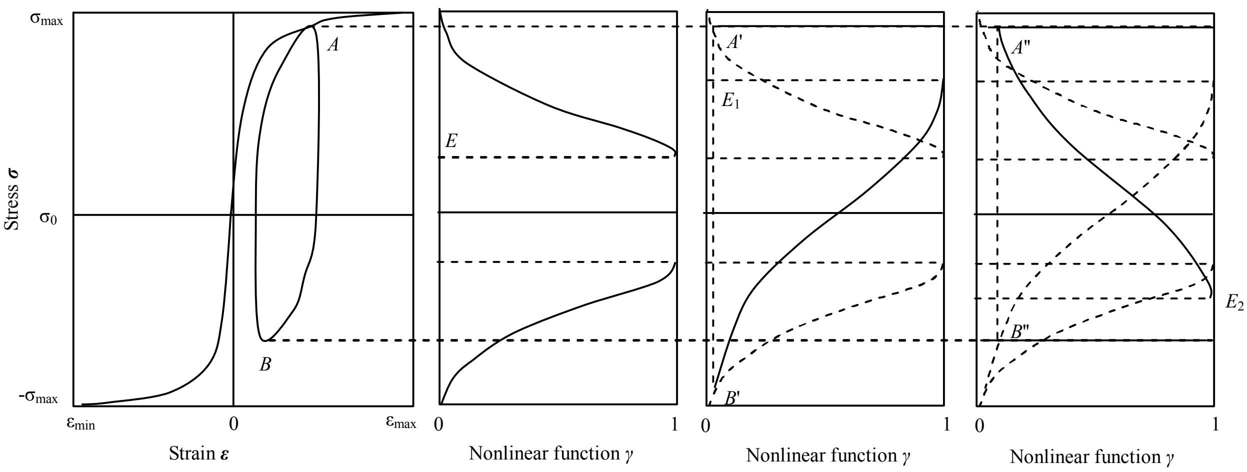
The flow rule is assumed to be associative

$$\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}=\frac{1}{H}(\boldsymbol{n}\mathrm{d}\boldsymbol{\sigma})\boldsymbol{n} \quad , \tag{20}$$

where $\boldsymbol{n}$ is the unit tensor collinear to the flow and yield directions ($\boldsymbol{n}$:$\boldsymbol{n}$=1), the plastic modulus H can be selected as the following function

$$H=\frac{H_0}{1-\gamma(\boldsymbol{\sigma})} \quad , \tag{21}$$

where H_0 is the plastic modulus in perfectly plastic state, and $\gamma(\boldsymbol{\sigma})$ ($0\leq\gamma\leq1$) is a nonlinear function of stress $\boldsymbol{\sigma}$.



(a) Stress-strain curve during compression, tension, and cyclic loading (b) Nonlinear function γ during compression and tension (c) Nonlinear function γ during unloading (d) Nonlinear function γ during reloading

Fig. 5. Fuzzy plastic model with purely elastic region

Each one-dimensional partition point is generalized into a yield surface. Each partitioning surface has a constant plastic modulus and a constant value of the nonlinear function γ. That is, on surface i, γ takes a constant value γ_i and H takes a constant value $H_0/(1-\gamma_i)$. For monotonic loadings, the largest partitioning surface ($i = N$) is the ultimate yield surface, whereas the smallest one ($i = 1$) is reduced to the initial stress state.

4.1 Fuzzification of stress $\boldsymbol{\sigma}$

N fuzzy sets are used. Each fuzzy set, denoted by $A_j(j=1,2,\cdots,N)$, has a continuous membership function μ_j . We use N partitioning surfaces to divide the stress space. At the initial stress state (i.e., $i = 1$), $\gamma=1$, $H\to\infty$, Eq. (19) predicts a purely elastic response. At the largest surface (i.e., $i = \boldsymbol{N}$), $\gamma=0$, $H=H_0$, Eq. (19) predicts a perfectly plastic response. The plastic modulus H and function γ between two adjacent partitioning surfaces are determined by a membership function.

4.2 Fuzzy rules

Rule 1: IF $\boldsymbol{\sigma}$ is $\boldsymbol{A}_1$ THEN $\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}=\frac{1-\gamma_1}{H}(\boldsymbol{n}\mathrm{d}\boldsymbol{\sigma})\boldsymbol{n}$ with the membership value w_1.

Rule 2: IF $\boldsymbol{\sigma}$ is $\boldsymbol{A}_2$ THEN $\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}=\frac{1-\gamma_2}{H}(\boldsymbol{n}\mathrm{d}\boldsymbol{\sigma})\boldsymbol{n}$ with the membership value w_2.

⋮

Rule j: IF $\boldsymbol{\sigma}$ is $\boldsymbol{A}_j$ THEN $\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}=\frac{1-\gamma_j}{H}(\boldsymbol{n}\mathrm{d}\boldsymbol{\sigma})\boldsymbol{n}$ with the membership value w_j.

⋮

Rule N: IF $\boldsymbol{\sigma}$ is $\boldsymbol{A}_N$ THEN $\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}=\frac{1-\gamma_N}{H}(\boldsymbol{n}\mathrm{d}\boldsymbol{\sigma})\boldsymbol{n}$ with the membership value w_N,

where $\gamma_j\,(0\le\gamma_j\le1)$ are design parameters.

4.3 Defuzzification

The centroid defuzzifier is used

$$\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}=\frac{\sum_{j=1}^{N}w_j[\frac{1-\gamma_j}{H}(\boldsymbol{n}\mathrm{d}\boldsymbol{\sigma})\boldsymbol{n}]}{\sum_{j=1}^{N}w_j}\ . \tag{22}$$

5 Fuzzy Plastic Model for Unsymmetric Cyclic Loadings

As previously stated, the plastic modulus H and function γ should be calculated on each loading reversal. This is easy for symmetric stress cycles. However, it may be difficult with unsymmetric stress cycles. We propose the following fuzzy plastic model to solve this problem by means of enlarging the partitioning interval.

5.1. Unloading

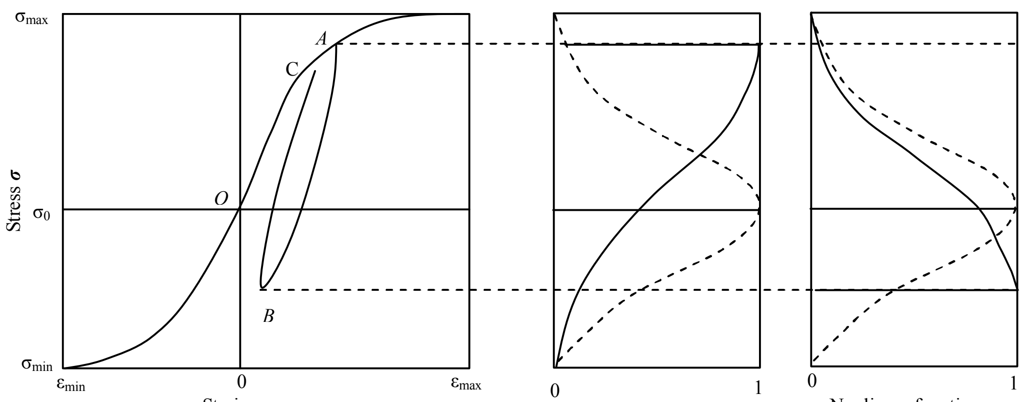
During the unloading from point A shown in Fig. 6(a), γ should represent the elastic response (i.e., $H\to\infty$, and $\gamma=1$) at the beginning of the unloading (i.e., $\boldsymbol{\sigma}=\sigma_A$). The partitioning stress space is enlarged to the interval $[\sigma_A, \sigma_{\min}]$. At the stress point $\sigma_{\min}$, we have $\gamma=0$, $H=H_0$, and this corresponds to the perfectly plastic response. It is very useful from the numerical point of view to transfer the stress $\boldsymbol{\sigma}=\sigma_x$ on the interval $[\sigma_A, \sigma_{\min}]$ into a dimensionless λ on the interval [0, 1] by

$$\lambda=\frac{\sigma_x-\sigma_A}{\sigma_{\min}-\sigma_A}\ . \tag{23}$$

When $\sigma_x=\sigma_A$, we have $\lambda=0$, $\gamma=1$, $H\to\infty$. When $\sigma_x=\sigma_{\min}$, we have $\lambda=1$, $\gamma=0$, $H=H_0$. As shown in Fig. 7.

5.2 Reloading

During the reloading from point B shown in Fig. 6(a), γ should represent the elastic response (i.e., $H\to\infty$, and $\gamma=1$) at the beginning of the reloading (i.e., $\boldsymbol{\sigma}=\sigma_B$). The partitioning stress space is enlarged to the interval $[\sigma_B, \sigma_{\max}]$. At the stress point $\sigma_{\max}$, we have $\gamma=0$, $H=H_0$, and this corresponds to the perfectly plastic response. In the same way, we transfer stress $\boldsymbol{\sigma}=\sigma_x$ on the interval $[\sigma_B, \sigma_{\max}]$ into a dimensionless λ on the interval [0, 1] by



(a) Stress-strain curve during compression, tension, and cyclic loading (b) Nonlinear function γ during unloading (c) Nonlinear function γ during reloading

$$\lambda = \frac{\sigma_x - \sigma_B}{\sigma_{\max} - \sigma_B} \quad . \tag{24}$$

When $\sigma_x = \sigma_B$, we have $\lambda = 0$, $\gamma = 1$, $H \to \infty$. When $\sigma_x = \sigma_{\max}$, we have $\lambda = 1$, $\gamma = 0$, $H = H_0$. So we can have the same fuzzy interval partitioning, as shown in Fig. 7.

Fig. 6. Fuzzy plastic model unsymmetric cyclic loadings

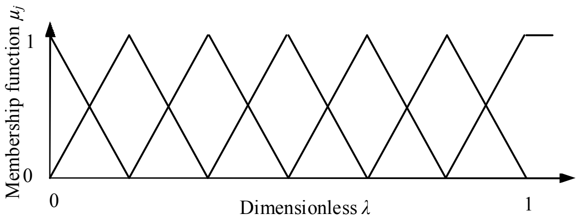


Fig. 7. Membership function

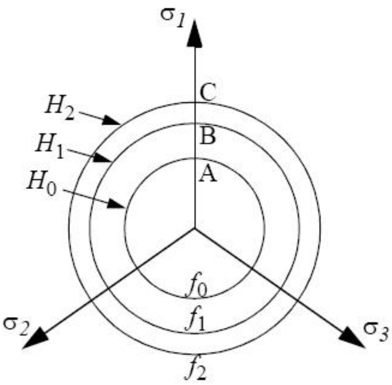
Similar to Monotonic Loadings, we can get

$$\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}} = \frac{\sum_{j=1}^{N} w_j \left(\frac{1-\gamma_j}{H_0} \mathrm{d}\boldsymbol{\sigma}\right)}{\sum_{j=1}^{N} w_j} \quad . \tag{25}$$

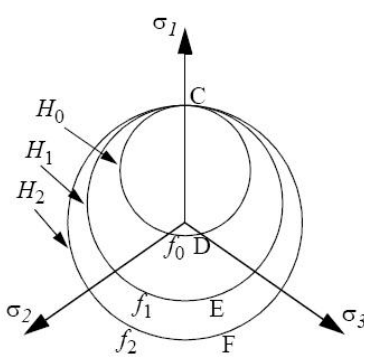
A similar approach can be used to model monotonic loadings and cyclic loadings with symmetric stress cycles.

6 Comparison with Multi-Yield-Surface Plasticity

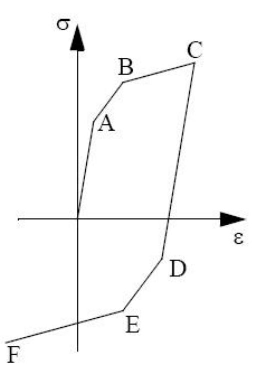
Multi-yield-surface plasticity for cyclic loading was originally proposed by IWAN[6] and MROZ[4]. In the case of tension-compression, the model uses the approximation of the stress-strain curve by linear segments with different hardening moduli. As shown in Fig. 8, in the stress space the multiaxial generalization is obtained by using a series of hypersurfaces $f_0, f_1, \cdots, f_m$, where f_0 is the initial yield surface and $f_1, f_2, \cdots, f_m$ are separate regions of constant hardening moduli[7]. The multi-yield-surface model provides a piecewise linear approximation of stress-stain curves, with an abrupt change in slope. The higher the number m of yield surfaces, the smoother the piecewise linear stress-strain curve of the multi-yield-surface model becomes, and the more accurate the simulation of the actual measured response could become[17]. When $m \to \infty$, the plastic modulus of the multi-yield-surface model converges toward a smooth distribution. This continuous distribution of H would provide a very accurate response of the material, but this would be impractical. In the fuzzy plastic model however, it can be approximated by a finite number of fuzzy partitioning surfaces. The passage from one model to another has been smoothened using fuzziness to prevent sudden changes in the model. Therefore the fuzzy plastic model can provide with a more realistic and practical description of stress-strain curves.



(a) Surfaces in the deviatoric plane before loading



(b) Surfaces at the loading point C



(c) Multilinear stress- strain curve[26]

Fig. 8. Multi-yield-surface plasticity by Mroz[4]

7 Comparison of Theoretical Prediction with Experiment

The capability of the fuzzy plastic models is investigated by comparing the predicted and experimental stress-strain responses of steels (tests done by OLSSON[20] and GOZZI[26]). The parameters of the fuzzy plastic model are calibrated from the experimental results. The least square identification method is used to search the optimum parameters of the fuzzy plastic model. Here, we use 6 membership functions to divide the stress space. The performances of the fuzzy plastic model and a two-surface model by GRANLUND[19] and OLSSON[20] are also compared. Fig. 9 shows the comparison between the

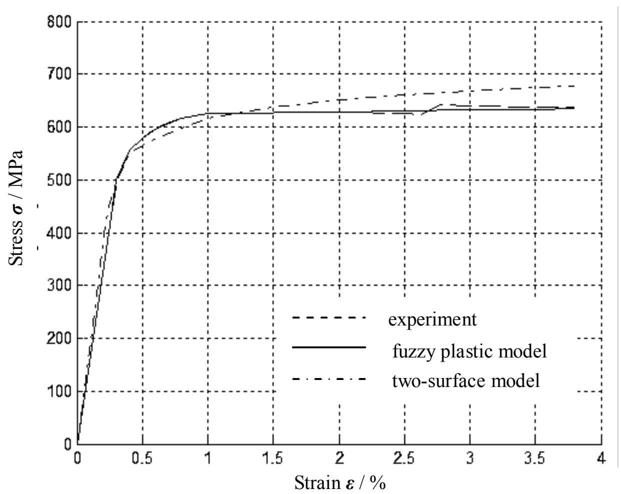
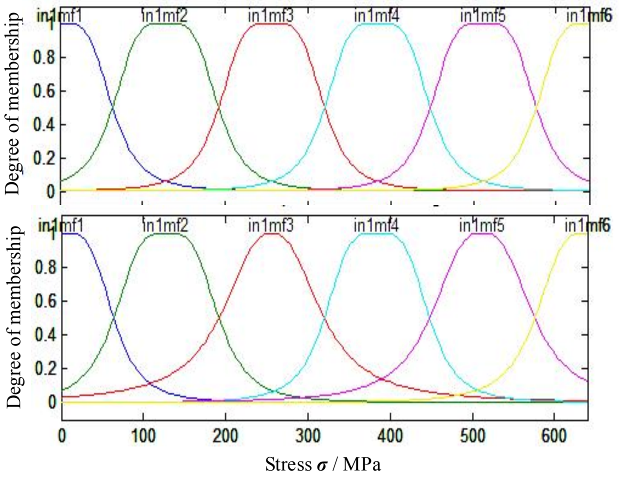


Fig. 9. Predicted stress-strain relation and a uniaxial test of the stainless steel 1.4318 C850 (Experimental data after

predicted and the experimental stress-strain responses for a stainless steel 1.4318 C850 in uniaxial test (experiment after GOZZI[26]). Fig. 10 shows the membership functions before and after training corresponding to the fuzzy plastic model of the stainless steel 1.4318 C850 during uniaxial loading. Fig. 11 shows the comparison between the predicted and the experimental stress-strain responses for a structural steel Weldox 1100 in uniaxial test (experiment after GOZZI[26]). As can be seen the overall agreement between the experimental results and the predictions by the fuzzy plastic model is very good. The predictions by the two-surface model[20, 26] are acceptable, but are of less accuracy.



(b) Membership functions after training

Fig. 10. Membership functions for the fuzzy plastic model of the stainless steel 1.4318 C850 during uniaxial loading

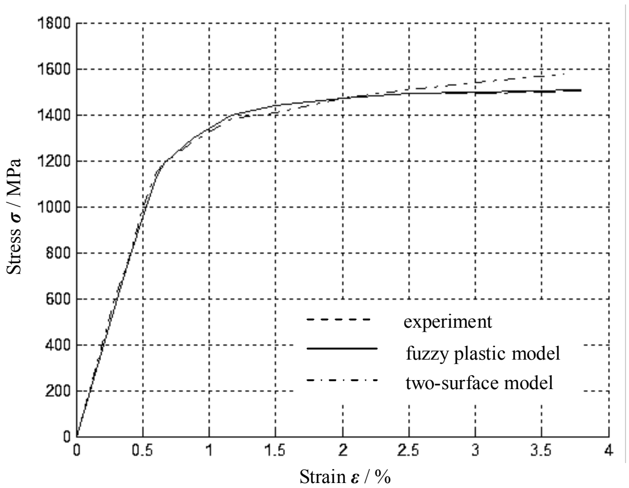


Fig. 11. Predicted stress-strain relation and a uniaxial test of the structural steel Weldox 1100 (Experimental data after GOZZI[26])

Fig. 12 shows the comparison between the predicted and the experimental effective stress–effective plastic strain curves for a stainless steel 1.4318 C850 in biaxial test (experiment after GOZZI[26]). Fig.13 shows the comparison of a structural steel Weldox 1100 in biaxial test (experiment after GOZZI[26]). The biaxial tests were carried out by OLSSON[20] and GOZZI[26] according to the following loading paths: each specimen was initially loaded in the same direction in the principal stress plane, unloaded and subsequently loaded in the initial loading direction plus 180^0. As can be seen the fuzzy plastic model can depict the response from the tests precisely, but the predictions by the two-surface model [20, 26] are not. The results show clearly the capabilities of the fuzzy plastic model.

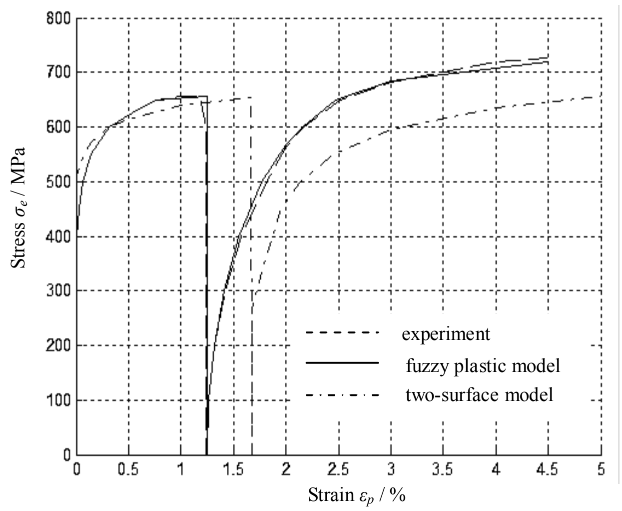


Fig.12. Predicted stress-strain relation and a uniaxial test of the stainless steel 1.4318 C850 (Experimental data after GOZZI[26])

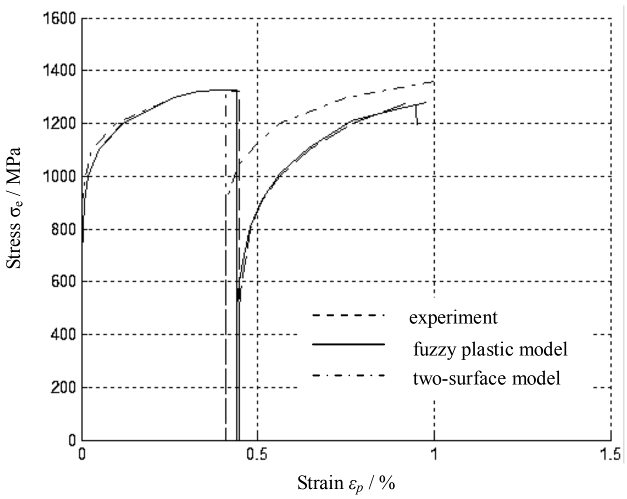


Fig.13. Predicted stress-strain relation and a uniaxial test of the structural steel Weldox 1100 (Experimental data after GOZZI[26])

8 Conclusions

(1) On the basis of the theory of fuzzy sets and

TAKAGI-SUGENO fuzzy model, a general fuzzy plastic model is developed. The proposed model uses a set of surfaces to partition the stress space, with each surface having a constant plastic modulus. The plastic modulus between two adjacent surfaces is determined by a membership function.

(2) The fuzzy plastic model can provide with a more realistic and practical description of the materials behavior than the classical plasticity model.

(3) The comparison between the predicted and the experimental stress-strain responses of steels is implemented. The results verify the capability of the fuzzy plastic model to simulate the stress-strain responses of materials during both monotonic and cyclic loadings.

(4) The fuzzy plasticity has the ability to handle many practical problems that cannot be adequately analyzed by the conventional theory of plasticity.

References

[1] PRAGER W. Recent developments in the mathermatical theory of plasticity[J]. *Journal of Applied Physics*, 1949, 20(3): 235–241.

[2] PRAGER W. A new method of analyzing stresses and strains in work-hardening plastic solids[J]. *Journal of Applied Mechanics*, 1956, 23(4): 493–496.

[3] ZIEGLER H. A modification of Prager's hardening rule[J]. *Quarterly of Applied Mathematics*, 1959, 17(1):55–65.

[4] MROZ Z. On the description of anisotropic workhardening[J]. *Journal of the Mechanics and Physics of Solids*, 1967, 15(3): 163–175.

[5] MROZ Z. On generalized kinematic hardening rule with memory of maximal prestress[J]. *Journal of applied mechanics*, 1981, 9(5): 241–259.

[6] IWAN W D. On a class of models for the yielding behavior of continuous and composite systems[J]. *Journal of Applied Mechanics*, 1967, 34(3): 612–617.

[7] CHABOCHE J L. Time independent constitutive theories for cyclic plasticity[J]. *International Journal of Plasticity*, 1986, 2(2): 149–188.

[8] DAFALIAS Y F, POPOV E. A model of nonlinearity hardening materials for complex loading[J]. *Acta Mechanica*, 1975, 21(3): 173–192.

[9] DAFALIAS Y F, POPOV E. Plastic internal variables formalism of cyclic plasticity[J]. *Journal of Applied Mechanics*, 1976.43(4): 645–651.

[10]DAFALIAS Y F. Bounding surface plasticity. I: mathematical foundation and hypoplasticity[J]. *Journal of Engineering Mechanics*, 1986, 112(9): 966–987.

[11]KRIEG R D. A practical two surface plasticity theory[J]. *Journal of Applied Mechanics*, 1975, 42(3): 641–646.

[12]HASHIGUCHI K. Constitutive Equation of elastoplastic materials with elastic-plastic transition[J]. *Journal of Applied Mechanics*, 1980, 47(1): 266–272.

[13]MROZ Z, NONIS V A. *Elasto-plsstic and viscoplastic constitutive models for soils with application to cyclic loading*[M]// PANDE G N, ZIENKIEWICZ O C. Soil mechanics-transient and cyclic loads. London: John Wiley and Sons. Lnc, 1982: 173–217.

[14]WACK B, TERRIEZ J M, GUELIN P. A hereditary type, discrete memory, constitutive equation with application to simple geometries[J]. *Acta Mechanica*, 1983. 50(1): 9–37.

[15]HASHIGUCHI K. Mechanical requirements and structures of cyclic plasticity models[J]. *International Journal of Plasticity*, 1993. 9(6): 721–748.

[16]BARDET J P. Scaled memory model for cyclic behavior of soils[J]. *Journal of the Geotechnical Engineering*, 1995, 121(11): 766–775.

[17]BARDET J P. Scaled memory model for undrained behavior of anisotropic clays[J]. *Journal of the Geotechnical Engineering*, 1995, 121(11): 755–765.

[18]BARDET J P. Scaled memory description of hysteretic matcrial behavior[J]. *Journal of Applied Mechanics,* 1996, 63(3), 750–757.

[19]GRANLUND J. Structural steel plasticity – experimental study and theoretical modeling[D]. Lulea: Lulea University of Technology, 1997:24. ISRN: LTU-DT-97/24-SE.

[20]OLSSON A. Stainless Steel plasticity – material modeling and structural applications[D]. Lulea: Lulea University of Technology, 2001:19. ISRN: LTU-DT-01/19-SE

[21]KLISINSKI M. Plasticity theory based on fuzzy sets[J]. *Journal of Engineering Mechanics*, 1988, 114(4): 563–582.

[22]KLISINSKI M, ABIFADEL N, RUNESSON K, et al. Modelling of the behavior of dry sand by an elasto-plastic “fuzzy set” model[J]. *Computers and Geotecnics*, 1991, 11(3): 229–261.

[23]ZADEH L A. Fuzzy sets[J]. *Information and Control*, 1965, 8(3): 338–353.

[24]TAKAGI T, SUGENO M. Fuzzy Identification of Systems and its application to modelling and control[J]. *IEEE Transactions on Systems, Man, and Cybernetics*, 1985, 15(1): 116–132.

[25]CHAKRABARTY J. *Theory of plasticity*[M]. Singapore: McGraw-Hill Book Company, 1988.

[26]GOZZI J. 2004. Plastic behaviour of steel: experimental investigation and modeling[D]. Lulea: Lulea University of Technology, 2004:51. ISRN: LTU-LIC-04/51-SE.

[27]ZHANG Liangchi. *Solid mechanics for engineers*[M]. Basingstoke, Hampshire: Palgrave, 2001.

[28]HILL R. *The mathemafical fheory of plasticity*[M]. Oxford: Oxford University Press, 1950.

[29]YING Hao. General SISO Takagi-Sugeno fuzzy systems with linear rule consequent are universal approximators[J]. *IEEE Transactions on Fuzzy Systems*, 1998, 6(4): 582–587.

[30]BESSELING J F. A theory of elastic, plastic and creep deformation of an initially isotropic material showing strain hardening creep recovery, and secondary creep[J]. *Journal of applied mechanics*, 1958, 25(4): 529–536.

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