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FURTHER REMARKS ON THE MODELLING OF ELASTIC MODULUS OF GRINDING WHEELS

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Abstract—A modified non-dimensional group for modelling the elastic modulus of grinding wheels is proposed and thoroughly discussed. Specific attention is paid to the temperature effect on the variations of the modulus. A new factor called processing parameter is successfully defined to reflect the behaviour of a wheel influenced by the manufacturing processes. The well constructed 3-D diagram shows that the variation of the non-dimensional elastic modulus is well represented by the present prediction and that the proposed method is convenient for both theoretical and engineering applications. In order to offer guidelines for engineers in drawing simple relations from the presented governing parameters, formulation procedures are clearly detailed through some illustrative examples.

NOMENCLATURE

a	empirical constant (equation (8))
b	empirical constant (equation (8))
C	empirical constant (equation (5))
d	grain diameter
d_{100}	grain diameter of grade 100
E	elastic modulus
$\bar{E}$	non-dimensional elastic modulus (equation (1))
E_0	elastic modulus at room temperature
$\bar{E}_0$	non-dimensional elastic modulus at room temperature
n	empirical constant (equation (5))
p	processing parameter (equation (3))
t	nominal time
T	temperature
T_0	room temperature
v_b	volumetric percentage of bond material
v_g	volumetric percentage of grain
V	total volume
X_1	non-dimensional parameter (equation (2))
X_{1Z}	non-dimensional parameter defined by Zhang et al. (equation (1))
X_2	non-dimensional parameter (equation (2))
X_{2Z}	non-dimensional parameter defined by Zhang et al. (equation (1))
X_3	non-dimensional parameter (equation (3))
ρ	specific weight
τ	a temperature related parameter proposed by Zhang et al. (equation (1))

1. INTRODUCTION

ELASTIC modulus of a grinding wheel is a key factor in determining the interference conditions between the wheel and a workpiece, which are characterized, macroscopically, by the contact length and the distributions of interface forces [1]. A knowledge of the interface conditions is, therefore, essential for the optimization of machining parameters for obtaining products with desired precision and surface finish. Although research has extensively been conducted from the viewpoint of elastic deformation of wheel–workpiece systems both microscopically and macroscopically [1–3], an open question is the relationship between the elastic modulus of a grinding wheel and some

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other physical quantities and manufacture conditions such as the overall density, grain size, grain type, as well as the manufacturing (e.g. baking) and working temperature.

Investigations into the elastic modulus have been mainly experimental, as summarized in Ref. [1]. In order to see the effect of a certain parameter such as the specific weight, one has to vary the value of that parameter but keep others unchanged. However, because of the number of parameters involved, the results have to be presented by means of a large number of curves of different series, which significantly detracts from an in-depth understanding of the dependence of elastic modulus upon key parameters. Very recently, based on a dimensional argument, Zhang et al. [4] successfully proposed a precise way of analysing the experimental data. Thus for a given non-dimensional temperature they were able to obtain only three dimensionless parameters. When plotting experimental data in this way, all the points collapsed into a single curve. Unfortunately, they did not discuss the details of the third dimensionless parameter and, in addition, the value of the second parameter may vary with the alteration of some other factors. The purpose of this paper is to offer a better dimensionless group based on the work of Zhang et al. [4]. The effect of temperature variation on the elastic modulus is carefully studied by introducing a judiciously defined processing parameter. A simple way to draw empirical formulae from the governing parameters is clearly presented.

2. ANALYSIS AND DISCUSSION

2.1. Modification of non-dimensional parameters

It has been revealed by Zhang et al. [4] that for a given type of grain and bond material the non-dimensional elastic modulus of a grinding wheel, $\bar{E}$, is only a simple function of two dimensionless variables:

$$\bar{E} = \Phi\left(X_{1Z}, X_{2Z}\right) , \tag{1}$$

where

$$\bar{E} \equiv \frac{E}{\rho d}, X_{1Z} \equiv v_g^3 v_b\left(\frac{d}{V^4}\right)^{-3/2} , X_{2Z} \equiv \tau ,$$

E is elastic modulus, d is grain mean diameter, ρ is specific weight, V is total apparent volume, v_g and v_b are volumetric percentages of grain and bond material, respectively, and τ is a temperature-related parameter. A unit volume V was considered throughout their calculations [4].

A weakness of equation (1) lies in that d/V^4 is not a very good definition as the numerical value may change with the unit of d used even though it appears dimensionless. A better way of overcoming this drawback, from the theory of dimensional analysis, is to use a normalized mean diameter with respect to any typical value of grain diameter. In fact, we could straight away use the diameter of grain of grade 100, d_{100}, to measure the grain diameter, and thus simply change equation (1) into:

$$\bar{E} = \Psi(X_1, X_2) , \tag{2}$$

where $X_1 \equiv v_g^3 v_b(d/d_{100})^{-3/2}$ and it becomes self-consistent.

The second variable in equation (1), X_2, involves the effect of temperature on the grinding wheel. The detailed definition of this variable was not emphasized [4]. It is well known that increasing the grinding temperature decreases the value of E even when other conditions of the wheel are the same. It is, therefore, convenient to define X_2 as the ratio of the work temperature of a wheel, T, to the room temperature, T_0; that is $X_2 = T/T_0$.

There is one more parameter which should be introduced into equation (2). It is the fact that different heating and cooling arrangements of manufacturing processes (for example, baking temperature and its duration) greatly influence the value of elastic modulus of grinding wheels, even when their components are the same. It is, therefore,

necessary to have a processing parameter to reflect such an effect and hence complete equation (2).

Let $t \in [0,1]$ be a nominal time which defines the manufacturing duration of a wheel in the numerically normalized interval $[0,1]$, and $f(t)$ be a processing function characterizing the process, mainly the baking process, in which the wheel is made. $f(t)$ is dimensionless and its distribution is controlled by wheel manufacturers. A processing parameter, p, can then be defined as:

$$p \equiv \int_0^1 f(t)\mathrm{d}t \ , \tag{3}$$

which is dimensionless and reflects the influence of manufacturing processes. Usually, for a specific manufacturer the distribution of $f(t)$ is the same for a given type of wheels, and therefore, the value of p is a constant. But the p value may vary from one manufacturer to another or from case to case when $f(t)$ is changed. Hence, a modified complete relation of equation (2) is given by:

$$\overline{E} = F(X_1, X_2, X_3) \ , \tag{4}$$

where

$$X_1 \equiv v_g^3 \, v_b(d/d_{100})^{-3/2}, X_2 \equiv T/T_0, \text{ and } X_3 \equiv \mathrm{p} \ .$$

2.2. Illustration and discussion

Equation (4) has been obtained based on the physics of the problem and on a dimensional argument. The functional dependence among the parameters has to be determined by a number of experimental results. For example, to see the effect of X_1 on $\overline{E}$ one should fix the values of X_2, and p. For practical purposes, it is always useful to demonstrate this method and to obtain empirical formulae based on existing experimental data.

The data in Refs [5–8] are re-plotted in the present way as shown in Fig. 1. They were all obtained at room temperature ($\sim25°C$). It can be seen that all the points broadly fall into one single curve, which justifies our argument in equation (4). This plot is similar to Fig. 2 in Ref. [4], but with a merit of self-dimensional consistency. Furthermore, the relationship between the dimensionless parameters $\overline{E}$ and X_1 may approximately be given by

$$\overline{E}_0 = C X_1^n \tag{5}$$

where the subscript "0" denotes the value at room temperature; C and n are numerical
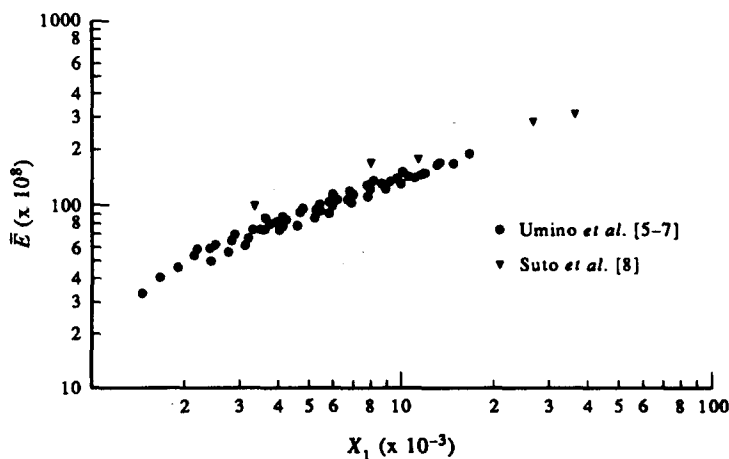

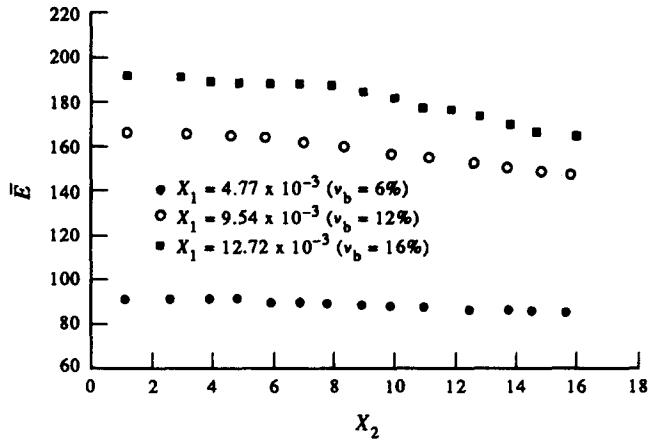Fig. 1. Dimensionless elastic modulus at room temperature.

FIG. 2. Dimensionless elastic modulus against temperature for different values of X_1.

constants. Using the least squares method, the calculated values of C and n are 2.95 × 10^{11} and 0.66, respectively.

The dimensionless equation (5) may also be re-written explicitly to predict the usual dimensional elastic modulus at room temperature, that is:

$$E_0 = C(d_{100})^{3n/2} \rho \, v_g^{3n} \, v_b^n \, d^{1-3n/2} \tag{6}$$

or

$$E_0 = 2.95 \times 10^{11} \, (d_{100})^{0.99} \rho \, v_g^{1.98} \, v_b^{0.66} \, d^{0.01} \tag{7}$$

with the particular values of C and n obtained above. This indicates that the dimensional elastic modulus under room temperature is strongly dependent upon ρ, v_g and v_b, but is a weak function of grain diameter d.

In order to see the dependence of $\overline{E}$ upon X_2 in equation (4), we need to fix the value of X_1. Figure 2 shows a dimensionless plot of the experimental data from Ref. [9] for three different values of X_1. It can be seen that as X_2 increases from the room temperature $\overline{E}$ decreases, particularly for wheels with large v_b. For the sake of obtaining a simple empirical formula, it is convenient to normalize all the data of $\overline{E}$ with respect to its value at room temperature, $\overline{E}_0$, corresponding to $X_2 = 1$. The results are shown in Fig. 3. It may be said that all the points fall more or less into one single curve, though the points for large values of X_2 are slightly scattered.

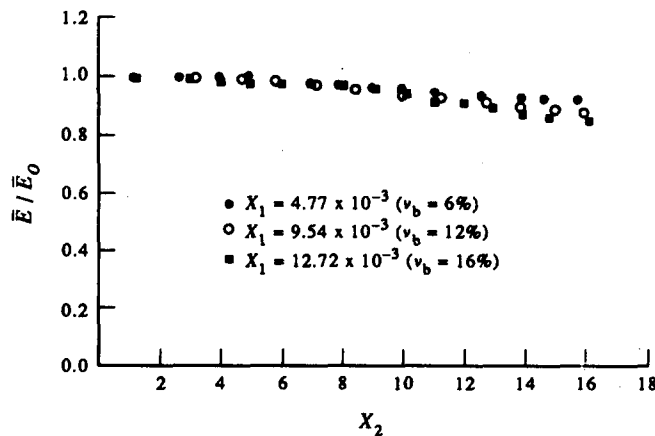All the points in Fig. 3 may be fitted using a formula of the following form:

FIG. 3. Variation of $\overline{E}/\overline{E}_0$ with X_2.

$$\overline{E}/\overline{E}_0 = 1 - a(X_2-1)^b , \tag{8}$$

where a and b are constants and are worked out to be 1.13×10^{-3} and 1.73, respectively. In general, it can be seen that when the temperature increases from room temperature to, say, 400°C, the value of $\overline{E}$ decreases by about 13%. In practice, temperature distribution over a grinding zone is not uniform as discussed by Zhang et al. [1]. The value of elastic modulus will therefore be reduced locally, which will in turn change, microscopically, some interfacial interaction factors between the wheel and the workpiece, such as the depth of grain cut.

Combining equations (5) and (8), we obtain the following general expression:

$$\overline{E} = \overline{E}_0[1 - a(X_2 - 1)^b] , \tag{9}$$

or

$$\overline{E} = C X_1^n [1 - a(X_2 - 1)^b] . \tag{10}$$

The functional dependence of $\overline{E}$ upon X_3, the process parameter p defined by equation (3), is not easy to determine. The easiest way is to assume $\overline{E} \propto p$. Thus by comparing the value of E_0 for nominally identical wheels for two manufacturers, one can obtain the ratio of the values of p of these two manufacturers. Accordingly the application of the data from Refs [5–8] and [9] indicate that the values of $\overline{E}_0$ from the latter are about 2.4 times those from the former. This suggests that under the present hypothesis the values of p differ by a factor of 2.4 in these two cases. Figure 4 provides a perspective view of the relationship among the three dimensionless variables, where the values of $\overline{E}$ from Ref. [9] have been divided by a factor of 2.4. It is obvious that more experimental studies need to be carried out in order to assess the effect of high temperature on the elastic modulus. Nevertheless, the present plot shows a promising prediction surface that presents the importance of the three governing non-dimensional variables. Clearly, further data will extend the surface to cover wider regimes. Consequently, this diagram successfully furnishes and summarizes the studies on the properties of the elastic modulus of grinding wheels and it offers a simple yet precise guideline for engineering practice.

3. CONCLUSIONS

(1) For a given type of grain and bond material, more reasonable dimensionless variables have been proposed and thoroughly discussed, which relate the elastic modulus at room temperature to other physical quantities such as the specific weight and the size of grains. The effect of temperature has been studied with the aid of a process parameter that has been defined to reflect the influence of manufacturing processes.
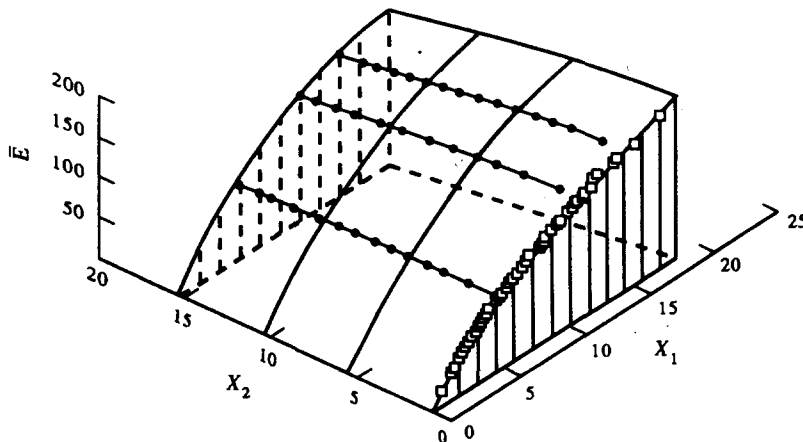

FIG. 4. The relationship among $\overline{E}$, X_1 and X_2.

The method provides engineers and researchers with a simple way of analysing experimental data. The constructed 3-D prediction surface enhances the studies of this topic.

(2) Based on the experimental data available for room temperature, an empirical formula has been found as follows:

$$E_0 = 2.95 \times 10^{11} \, (d_{100})^{0.99} \, \rho \, v_g^{1.98} \, v_b^{0.66} \, d^{0.01} \ . \tag{11}$$

(3) High temperature reduces the value of the elastic modulus. It has been suggested that the elastic modulus at a high work temperature T may be related to that at room temperature T_0 in the following way:

$$E = E_0 \left[1 - 1.13 \times 10^{-3} \left(\frac{T}{T_0} - 1 \right)^{1.73} \right] \ . \tag{12}$$

(4) The above results offer a direct guideline for further theoretical studies and engineering applications. Any of equations (10), (11) and (12) may easily be used when detailed variations of the elastic modulus of a grinding wheel in the grinding zone should be taken into account, such as in the study of micro-cutting of individual grains.

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