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Assessment of constitutive equations used in machining

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Abstract. Machining of metals is characterised by plastic deformation occurring at high strains, strain-rates and temperatures. In the few predictive machining theories reported in literature, two types of constitutive equations, the power-law relation and Johnson-Cook equation, have mainly been used for describing the material's plastic behaviour. This paper aims at assessing the above constitutive equations in terms of their ability to describe the material behaviour for a wide range of strain-rates and temperatures encountered in machining. The focus is on the plain carbon steel and copper work materials. Using the above constitutive equations, flow stress results were determined for conditions encountered in machining and then compared with experimental results obtained from literature which are compatible with the rate equations applicable to the microscopic deformation mechanisms under considered conditions. It was concluded that considerable improvements are required for the aforementioned constitutive equations for accurate prediction of the work material behaviour under machining conditions and that further development is needed for more reliable and accurate constitutive equations.

Introduction

The mechanics of machining investigates the deformation of a workpiece subjected to machining, such as chip formation, machining forces, residual stresses, temperature effects, strain rate effects and surface integrity. An important aspect of the investigation is to provide an in-depth understanding of the theoretical relationships for cutting forces, temperatures, stresses, etc in terms of work material properties, tool geometry and machining conditions. These relationships are then used to predict process parameters such as power requirements and tool life.

During a chip formation process plastic deformation mainly occurs in two regions. These are the primary deformation zone where the chip is formed and secondary deformation zone which arises because of frictional conditions at the tool chip interface. In analytical studies of machining, deformation processes occurring at these plastic zones are quantitatively analysed. One of the major difficulties encountered in these studies is the lack of suitable constitutive equations to describe accurately the variations of flow stress with strain, strain-rate and temperature and the lack of data for the extreme conditions of strain, strain rate and temperature encountered in machining. For example, during machining of plain carbon steels under practical conditions, typical values of strains are 1-2 in the primary deformation zone and above 3 in the secondary deformation zone, while strain rates are 10^3 - 10^6 s^{-1} in both zones and temperatures are 200 to 400 °C in the primary deformation zone and 800-1200 °C at the tool chip interface [1]. Despite the above difficulties, commendable attempts for developing predictive methods have been reported. In these predictive methods, two of the commonly used constitutive equations are the empirical or power law stress strain relation and Johnson-Cook equation. The power-law relation has so far been applied for plain carbon steel [1] and aluminium alloy [2] work materials. Johnson-Cook equation [3] has been applied for the above two work materials and hardened steel.

For the high strain-rates encountered in machining, the microscopic deformation mechanisms involved are best described by plasticity limited by electron and phonon drags and relativistic effects at low temperatures and power law break down at high temperatures [4]. Approximate rate equations that would describe the above deformation mechanisms are also given in [4]. However, as pointed out by Harding [5], a constitutive equation that would accurately describe the macroscopic plastic behaviour of a material while soundly based on the microscopic deformation processes is still some way from being achieved. Although more scientifically justifiable semi-empirical constitutive equations are available, the very extensive experiments required for them have compelled researchers to choose more empirically based relations such as those mentioned above.

The present paper aims at critically assessing the power-law and Johnson-Cook constitutive equations in terms of their ability to describe the material behaviour for a wide range of strain rates and temperatures encountered in machining. Due to the availability of the experimental data and empirical constants of these constitutive equations, focus of the present work is on the plain carbon steel and copper work materials. For these high strain rates, experimental flow stress results which are in agreement with the relevant rate equations (describing the deformation mechanisms involved) have been obtained from literature. These flow stress results are then compared with those predicted from the above constitutive equations under identical/similar conditions. Based on these comparisons, further research required for the development of more reliable constitutive equations for predictive machining theories are suggested. The following sections start with a review of the aforementioned two types of constitutive equations.

The power-law stress-strain relation

This type of equation was originally used in a machining theory for plain carbon steel work materials by Oxley and his co-workers [1, 6]. This relation is given by

$$\sigma = K\varepsilon^{n} \qquad (1)$$

where σ is uniaxial flow stress, ε is uniaxial strain, K is strength coefficient and n is strain hardening exponent. Both K and n are assumed to be functions of strain-rate and temperature. A brief review of the development of this equation is as follows.

Oxley used the flow stress data obtained by Oyane et al [7] for plain carbon steels (with 0.16%, 0.33%, 0.49% and 0.52% carbon) from high speed compression tests carried out at strain rate 450 s^{-1} for a temperature range 0-1100 °C and strains up to 1 or more. Although the strain-rates used in these tests were much lower than those encountered in machining (10^3 to 10^6 s^{-1}), the use of the velocity modified temperature concept allowed Oxley to extrapolate these flow stress data to the machining range. That is, for Eq.1, it was assumed that for a given strain ε, the flow stress for a particular material is a unique function of the velocity modified temperature T_{mod} defined as

$$T_{mod} = T\,[\,1 - \nu\, ln(\dot{\varepsilon}/\dot{\varepsilon}_{o})]$$

where T is absolute temperature, $\dot{\varepsilon}$ is the direct strain rate and ν and $\dot{\varepsilon}_{o}\,(=1\,s^{-1})$ are constants.

At a given temperature, for each of the plain carbon steels considered, results of Oyane et al [7] were used to determine K and n in Eq.1. The values of K and n thus obtained were then plotted against T_{mod} (with ν=0.09) to obtain curves representing the flow stress properties of each of the steel. By representing these curves mathematically and using rescaling functions, continuous changes in K and n over the ranges of T_{mod} and carbon content considered were represented by a set of functions. From these functions given in [1], the K and n curves obtained for plain carbon steels with 0.12% and 0.45% carbon are given in Fig.1. Oxley has used this type of curves in his machining theory for predicting the flow stress at given values of strains, strain-rates and temperatures.

Johnson-Cook constitutive equation

In this equation, the von Mises yield stress is expressed as [3]

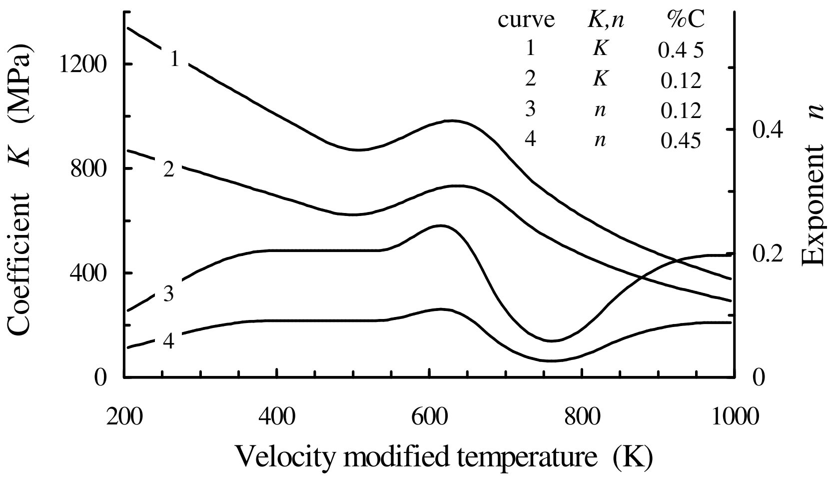

Fig.1 Variations of K and n with T_{mod} for 0.12% and 0.45% carbon steels

$$\sigma = \left(A + B\varepsilon^{n}\right)\left(1 + C\,ln\,\dot{\varepsilon}^{*}\right)\left(1 - T^{*m}\right) \qquad (2)$$

where $\dot{\varepsilon}^{*}$ is the ratio $\dot{\varepsilon}/\dot{\varepsilon}_{o}$ (with $\dot{\varepsilon}_{o} = 1\,s^{-1}$) and T^{*} is the ratio $(T\text{-}T_r)/(T_m\text{-}T_r)$ with T_r and T_m being absolute room and melting temperatures respectively. *A, B, C*, *n* and *m* are material constants. The expression in the first set of brackets represents power-law work hardening curve with *n* assumed to be constant. The expression in the second set of brackets represents the semi-logarithmic dependence of stress on strain rate while that in the third set of brackets represents the reduction in strength due to the increase in temperature (thermal softening) resulting from plastic work. Johnson and Cook [3] obtained these material constants for oxygen free high conductivity (OFHC) copper, armco iron and AISI-4340 steel. The required experimental data were obtained from torsion tests, Hopkinson bar tests and tensile tests. It is noteworthy that the highest strain-rate reached during these tests was 650 s^{-1}.

Thus it can be seen that the highest strain rates tested in [3, 7] were well below 1000 s^{-1}. The results from these tests were used to determine the constants of Eq.1 and Eq.2 which were subsequently applied at very high strain rates (10^3 to 10^6 s^{-1}). Hence the present work now investigates the predictive capability of the above constitutive equations at these high strain rates.

Plastic deformation at high strain-rates

As pointed out by Frost and Ashby [4], crystalline solids deform plastically by a number of alternative, often competing deformation mechanisms. For machining of metals under practical conditions, where strain rates are in the range 10^3 - 10^6 s^{-1}, the relevant deformation mechanisms are phonon and electron drags and relativistic effects at low temperatures and power law breakdown at high temperatures. While these authors have given approximate model based rate equations that would describe the above deformation mechanisms, they have also pointed out that the major difficulty encountered is the non-availability of appropriate experimental data. Utilising the published experimental data for copper, aluminium and titanium, they have shown that, at low temperatures[1], for a substantial portion (at the lower end) of the strain rate range 10^3 - 10^6 s^{-1}, a linear relationship between flow stress and strain-rate should exist due to the dominance of phonon and electron drags. However, at very high strain rates, eg. When approaching 10^6 s^{-1}, the above relationship appears to deviate from the linear one due to relativistic effects, etc [4]. Notably,

[1] In the present work, these low temperatures are assumed to be those below $0.5\times T_m$ where T_m is the absolute melting temperature of the material

Ferguson et al [8], Kumar et al [9], Kumar and Kumble [10] and Campbell and Ferguson [11] obtained experimental data at the lower end of the above strain rate range that shows a linear relationship between flow stress and strain rate. The work materials considered in the above investigations were mono-crystalline zinc [8], aluminium (mono-/poly-crystalline) [9], OFHC copper [10] and plain carbon steel with 0.12% carbon [11]. These experimental flow stress results are now compared with those predicted using the power-law relation (Eq.1) and Johnson-Cook equation (Eq.2) for materials for which required constants can also be found from literature.

Comparison between predicted and experimental results

For a plain carbon steel (0.12%C), Campbell and Ferguson [11] have determined the lower yield point using dynamic shear tests of Kolsky thin-wafer type for wide ranges of strain-rates (10^{-3} to 4×10^{4} s^{-1}) and temperatures (195-713 K). Using their experimental results for the high strain rates, the uniaxial flow stress and strain rate values have been calculated by the present authors. In these calculations, the von Mises yield criterion is assumed. The calculated results are depicted in Fig.2 by symbols with the thin lines representing the linear regression lines fitted to each set of experimental data corresponding to a particular temperature. It can be seen that Campbell and Ferguson's experimental results can be represented very well by a linear relationship which is also in agreement with the rate equations relevant to the applicable deformation mechanism mentioned above. The thick curves in Fig.2 represent the flow stress results predicted from Eq.1 using the corresponding curves for K and n given in Fig.1. It can be seen that the predicted results show a much lower rate of increase in flow stress with strain-rate than that indicated by the experimental results. The predicted variation of flow stress with temperature appears to be reasonable. However, for this plain carbon steel, it is not possible to compare the predicted flow stress results using Johnson-Cook equation (Eq.2) since the required constants are not available.

For plain carbon steel with 0.45% carbon, Jaspers and Dautzenberg [12] have determined the constants of the Johnson-Cook equation using the data obtained from their experiments. The experimental data at high strain rates (up to 7.5×10^{3} s^{-1}) were obtained from split Hopkinson pressure bar technique for temperatures up to 500 °C and uniaxial strains up to 0.3. The obtained constants of Eq.2 are: A=553.1 MPa, B=600.8 MPa, C=0.0134, n=0.234, m=1. Using these constants and Eq.2 (with T_m=1733 K), the present authors have predicted the flow stress results at high strain rates (5000 to 30,000 s^{-1}) and at strain 0.05 and temperatures 20, 220 and 440 °C. These results together with those predicted using the power-law relation (Eq.1) are given in Fig.3. For obtaining the latter results, the corresponding K and n curves given in Fig.1 were used. It can be seen that the difference between the flow stress results determined from Eq.1 and Eq.2 is greatest at low temperatures. These differences can be seen to decrease with increase in temperature. More importantly, both Eq.1 and Eq.2 seem to predict a very low rate of increase in flow stress with strain rate. However, based on the results in Fig.2 (compared to experimental results, Eq.1 was found to predict a lower rate of increase in flow stress with strain rate), Eq.1 and Eq.2 appear to considerably underestimate this rate of increase. Moreover, Eq.2 appears to predict a lower rate of increase in flow stress with strain rate than that predicted by Eq.1. This indicates possible greater underestimations of the flow stress from Eq.2 than from Eq.1. For the considered steel and strain rate range, the authors were unable to find experimental data to compare with the predicted results.

For annealed and work hardened OFHC copper, experimental flow stress results have been given by Kumar and Kumble [10] for uniaxial strains up to 0.1, temperatures in the range 300-590 K and strain rates up to 5000 s^{-1}. According to their data, for strain rates above 500 s^{-1}, a clear linear relationship can be seen between the flow stress and strain rate. As noted earlier, Johnson and Cook [3] have determined the constants of Eq.2 for OFHC copper as follows: A=90 MPa, B=292 MPa, C=0.025, n=0.31 and m=1.09. They have then used these constants for predicting flow stress for strain rates up to 10^{5} s^{-1}. Using these constants and Eq.2 (with T_m=1356 K), the present authors have predicted the flow stress results at high strain rates (500-5000 s^{-1}), temperature 300 K and strains 0.02 and 0.04, as depicted in Fig.4. It can be seen that the predicted stress versus strain rate relation deviate from the expected linear relation. A quantitative comparison between these predicted results

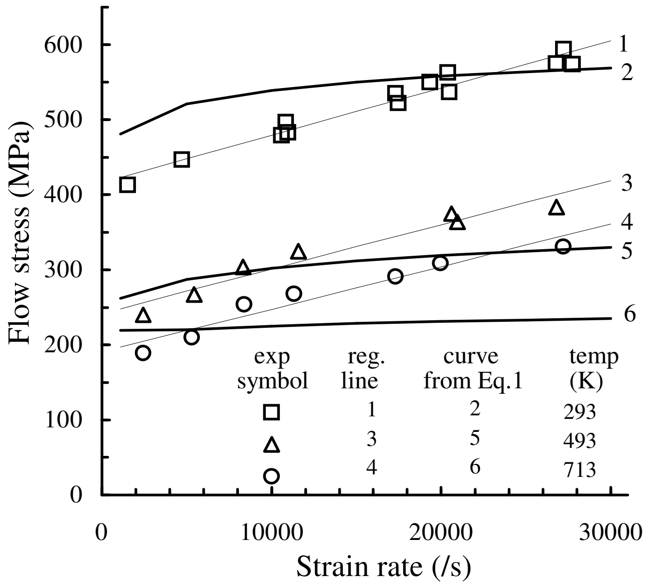

Fig.2. Variation of flow stress with strain rate and comparison between predicted and experimental results (ε=0.0058)

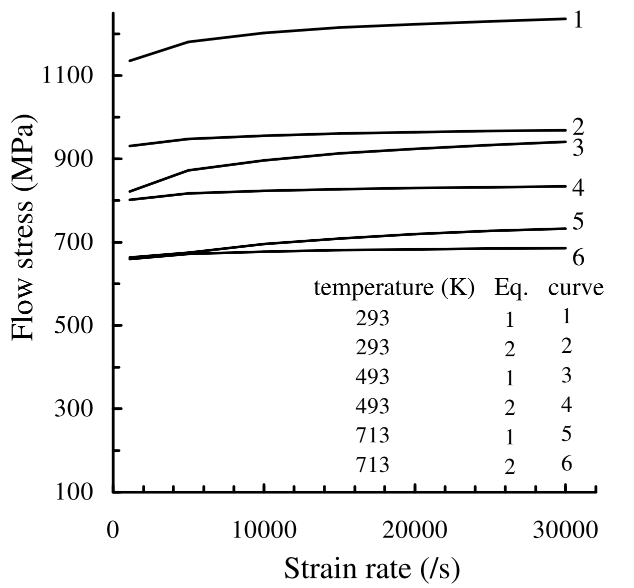

Fig.3. Predicted flow stress versus strain-rate results for 0.45% carbon steel (ε=0.05)

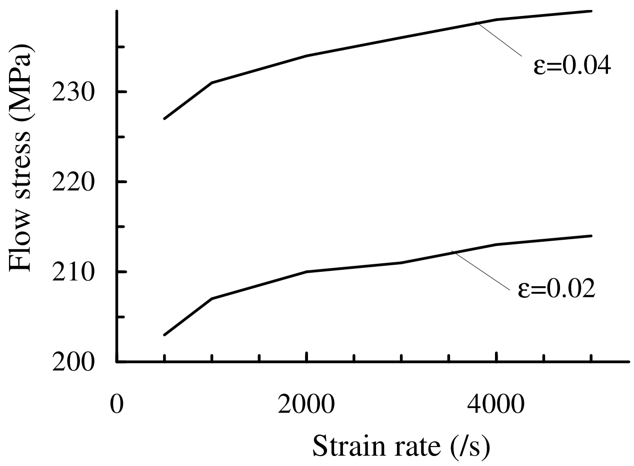

Fig.4. Predicted flow stress versus strain rate results for OFHC copper at temperature 300 K

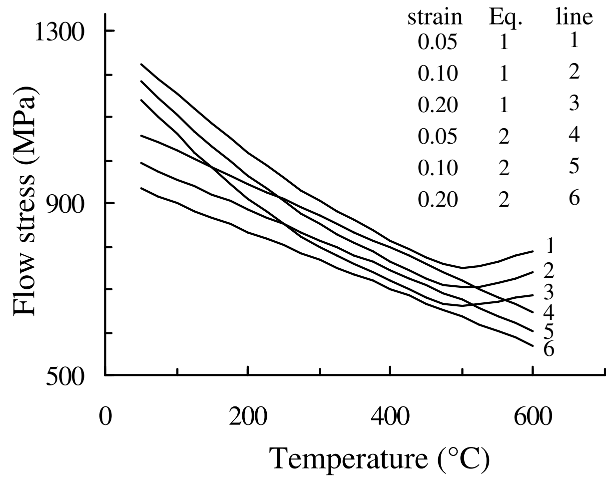

Fig.5. Predicted flow stress versus temperature results for 0.45%C steel at strain-rate 7500 s^{-1}

and experimental results (from [10]) is not possible since the present authors were not able to obtain the details of the heat treatment used for the copper materials considered in [10] and [3].

So far this paper has considered the predicted/experimental flow stress versus strain rate relation for the high strain rates applicable to plastic deformation in machining. When assessing the constitutive equations used in machining, it is also necessary to consider the relation between flow stress and strain and that between flow stress and temperature at these high strain rates. The major difficulty encountered in these considerations is the scarcity of the required experimental data. It was noted that Jaspers and Dautzenberg [12] have given the experimental flow stress results for 0.45% carbon steel at strain rate 7500 s^{-1} for strains in the range 0.05-0.2 and temperatures in the range 50-600 °C. For all strain values considered, their experimental results show a decrease in flow stress with increase in temperature up to ~500 °C. Above this temperature, flow stress shows an increase with increase in temperature[2]. For the above conditions, the present authors have predicted the flow stress versus temperature results using the power-law relation (Eq.1 with the corresponding

[2] This particular increase in flow stress with increasing temperature known as dynamic strain aging or blue brittleness is typical of plain carbon steels.

K and *n* curves from Fig.1) and Johnson-Cook equation (Eq.2). These results are shown in Fig.5. It can be seen that only Eq.1 can correctly predict the increasing trend of flow stress with increasing temperature above ~500 °C. Note that the experimental results given in [12] are not included in Fig.5 since it is very likely that these results were used by Jaspers and Dautzenberg for obtaining the empirical constants of Eq.2 which were used in determining the curves 4, 5 and 6 in Fig.5.

In this paper, for the high strain rates encountered in machining, comparisons between predicted and experimental flow stress results and those between predictions from Eq.1 and Eq.2 have been presented. These comparisons are not as comprehensive as the authors have intended because of the non-availability of the required experimental data[3] and/or empirical constants of the constitutive equations. Nevertheless, the presented results for plain carbon steels show that the power-law relation (Eq.1) is able to represent the experimentally observed variations of flow stress with strain-rates and temperatures more accurately than Eq.2. This is possibly due to (i) expression of *K* and *n* of Eq.1 as 7th order polynomials of T_{mod} which is function of temperature and logarithmic strain rate and (ii) comprehensive procedure used in determining the required empirical constants [1].

Conclusions

As discussed in this paper, for many metallic work materials, at low temperatures, substantial experimental evidence can be found from published literature that indicates a clear linear relationship between the flow stress and strain rate for the lower part of the strain rate range (10^3 - 10^6 s^{-1}) encountered in machining. This linear relationship has been explained using the microscopic deformation mechanism electron and phonon drag. The presented results for plain carbon steels show that the considered power-law relation (Eq.1) is able to represent the variations of flow stress with strain-rates and temperatures with better accuracy than Eq.2. However, the given results also show that considerable improvements are needed for these constitutive equations for accurate prediction of flow stress under machining conditions. It seems, despite considerable research during the last 60 years or so, there is no sufficient data, particularly for the very high strain rates and temperatures, which are required for the development of reliable constitutive equations suitable for predictive theories of machining.

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[3] Unfortunately for strain rates above 50000 s^{-1} and temperatures above $0.5\times T_m$ authors were unable to find reliable experimental data for the considered materials.

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