

INVESTIGATION OF SHEET METAL FORMING BY BENDING— PART III: APPLICABILITY OF DEFORMATION THEORY TO STAMPING OF CIRCULAR SHEETS

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Abstract—In this paper the applicability of J_2 deformation theory to the stamping of circular sheets is discussed through the comparison of various cases whose loading paths deviate from proportional loading paths to varying degrees. It is seen that the potential of deformation theory is not as great as has previously been supposed.

NOTATION

See Ref. [1].

1. INTRODUCTION

The authors investigated the stamping of circular sheets in the conical die cup test using simple J_2 flow theory and obtained satisfactory results in Parts I and II of this study [1, 2]. However, there exists a disadvantage: the method uses a lot of computer time. Although the maDR method makes it possible to solve complicated problems with microcomputers, it does affect the efficiency of so doing to various degrees. The main factor responsible for this weakness is the incremental algorithm of the flow theory. Therefore, researchers, and especially engineers, are continually searching for more convenient approaches.

Deformation theories are simple, and have received great attention for over half a century. Although not entirely satisfactory from the point of view of theoretical mechanics, their potential seems to be considerable, especially with reference to the plastic buckling paradox of plates and shells. The latter has caused many researchers to study again the general applicability of such theories. The discussion by Budiansky in Ref. [3] is excellent in this area, and showed that deformation theories of plasticity may be used for a range of loading paths other than those of proportional loading without violating the general requirements for the physical soundness of plasticity theory—i.e. he showed that deviations from proportional loading are admissible. Furthermore, he gave a *total loading path* to expand the range of application of deformation theory. However, investigations of specific problems seem to show that many more deviations than just total loading are possible. For example, the analyses of the deformation of a hollow cylinder subjected to internal pressure [4] and torsion [5], and the torsion-bending [6] of square bars and the bending of circular plates subjected to uniform transverse pressure [7], showed that solutions from deformation theory were in good agreement with those from flow theory and experimental results. BeiChuan Hao [8] has concluded that deformation theories are applicable as long as no local unloading occurs during the loading process, even if the loading path deviates far from proportional loading.

Can one, therefore, obtain satisfactory results from deformation theories for the stamping problems of sheet metals? If the answer is in the affirmative, it will make engineering analysis quite simple.

Let us first study the loading paths at some points of circular sheets 150-2.0-C45, 150-2.0-C65, 250-10.0-C70 and 250-10.0-UC with the aid of the simple J_2 flow theory, where UC means that the plate is subjected to a uniform pressure and is clamped on the periphery of its mid-plane, and other notation is explained in Refs [1, 2]. Figure 1 shows that the loading paths of the points in 250-10.0-UC are very close to those of proportional loading, while those in 250-10.0-C70 deviate from proportionality slightly, but those in 150-2.0-C45

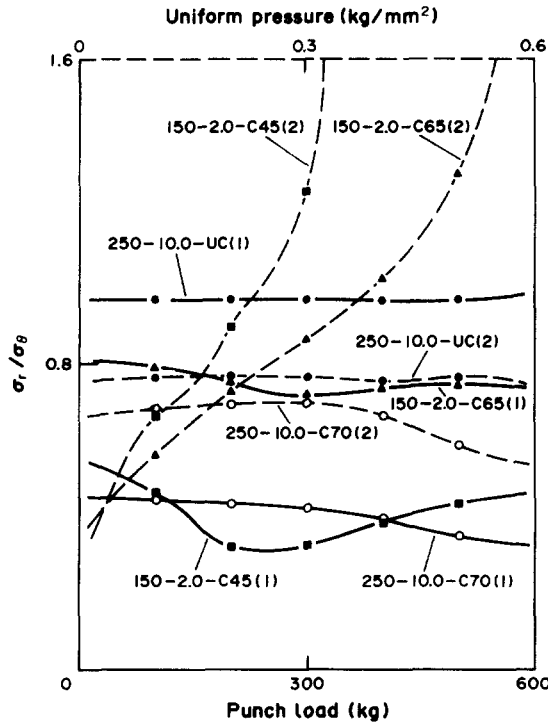


FIG. 1. Variation of σ_r/σ_θ of various plates (coordinates of the points (r, z) (mm)). 150-2.0-C45(1): (37.5, -1), 150-2.0-C45(2): (60, 1); 150-2.0-C65(1): (42.875, -1), 150-2.0-C65(2): (58.929, 1); 250-10.0-UC(1): (50, -5), 250-10.0-UC(2): (25, 5); 250-10.0-C70(1): (87.5, -5), 250-10.0-C70(2): (62.5, 5).

and 150-2.0-C65 deviate far from it. Moreover, the calculations show that no local unloading occurs during the whole loading processes of 250-10.0-UC and 250-10.0-C70. We can therefore examine in some detail the applicability of J_2 deformation theory to the various cases of the bending of circular plates.

2. SOLUTION BY J_2 DEFORMATION THEORY

2.1. Loading process

As shown in Fig. 2, the radial section of a circular plate is divided into four regions: region I is the elastic region, regions II and IV are those with single-sided plastic regions, and region III is that with a double-sided plastic region. The equilibrium equations (see equation (8) in [1]) are now simplified to

$$\left. \begin{aligned} \frac{\partial N_r}{\partial r} + \frac{1}{r}(N_r - N_\theta) &= 0, \\ \frac{\partial^2 M_r}{\partial r^2} + \frac{1}{r} \left(2 \frac{\partial M_r}{\partial r} - \frac{\partial M_\theta}{\partial r} \right) + N_r \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} N_\theta \frac{\partial w}{\partial r} + q &= 0 \end{aligned} \right\}, \quad (1)$$

where w is the vertical mid-plane displacement, q is the external uniform transverse pressure, N_r and N_θ are membrane forces and M_r and M_θ are the bending moments in the radial and circumferential directions, respectively. The geometrical relations (see equation (10) in [1]) can be expressed correspondingly as

$$\left. \begin{aligned} \varepsilon_r^0 &= \frac{\partial u}{\partial r} + \frac{1}{2} \left(\frac{\partial w}{\partial r} \right)^2, & \varepsilon_\theta^0 &= \frac{u}{r} \\ K_r &= -\frac{\partial^2 w}{\partial r^2}, & K_\theta &= -\frac{1}{r} \frac{\partial w}{\partial r} \end{aligned} \right\}, \quad (2)$$

and

$$\varepsilon_r = \varepsilon_r^0 + zK_r, \quad \varepsilon_\theta = \varepsilon_\theta^0 + zK_\theta$$

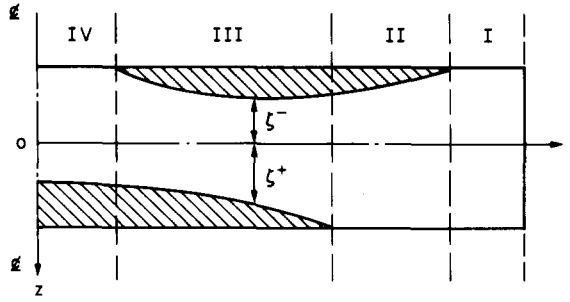


FIG. 2. Elastic and plastic regions in the radial section.

where u is the radial mid-plane displacement, ε_r and ε_θ are the strain components and K_r and K_θ are curvatures in the radial and circumferential directions, respectively.

In the elastic regions, Hooke's law applies, and can be written as

$$\left. \begin{aligned} \sigma_r^e &= \frac{E}{1-\nu^2} [\varepsilon_r^0 + \nu \varepsilon_\theta^0 + z(K_r + \nu K_\theta)] \\ \sigma_\theta^e &= \frac{E}{1-\nu^2} [\varepsilon_\theta^0 + \nu \varepsilon_r^0 + z(K_\theta + \nu K_r)] \end{aligned} \right\}, \quad (3)$$

while in the plastic regions, J_2 deformation theory gives

$$\left. \begin{aligned} \sigma_r^p &= \frac{2}{3} E_s \{ (1 + H_1^*) \varepsilon_r^0 + H_1^* \varepsilon_\theta^0 + z[(1 + H_1^*) K_r + H_1^* K_\theta] \} \\ \sigma_\theta^p &= \frac{2}{3} E_s \{ (1 + H_1^*) \varepsilon_\theta^0 + H_1^* \varepsilon_r^0 + z[(1 + H_1^*) K_\theta + H_1^* K_r] \} \end{aligned} \right\}, \quad (4)$$

where the superscripts e and p stand for elastic and plastic principal stress components, respectively, and

$$\left. \begin{aligned} H_1^* &= \frac{E/(1-2\nu) - 2E_s/3}{E/(1-2\nu) + 4E_s/3} \\ E_s &= \frac{3EE_s^0}{3E - (1-2\nu)E_s^0} \end{aligned} \right\}, \quad (5)$$

where E and ν denote the Young's modulus and Poisson's ratio, respectively, and E_s^0 is the secant modulus of the uniaxial stress-strain curve of the sheet material.

It is assumed that the plastic regions are monotonically developed during the loading process and that the boundary between the elastic and plastic regions is at $z = \zeta$. The continuity of stresses across the boundary and the Mises yield condition lead to the relation

$$A_1 \zeta^2 + A_2 \zeta + A_3 = 0, \quad (6)$$

where

$$\left. \begin{aligned} A_1 &= B_1(K_r^2 + K_\theta^2) + B_2 K_r K_\theta \\ A_2 &= \varepsilon_r^0 (2B_1 K_r + B_2 K_\theta) + \varepsilon_\theta^0 (B_2 K_r + 2B_1 K_\theta) \\ A_3 &= B_1(\varepsilon_r^{02} + \varepsilon_\theta^{02}) + B_2 \varepsilon_r^0 \varepsilon_\theta^0 + B_3 \end{aligned} \right\}, \quad (7)$$

and

$$\left. \begin{aligned} B_1 &= \nu^2 - \nu + 1, \\ B_2 &= -\nu^2 + 4\nu - 1, \\ B_3 &= -[Y(1-\nu^2)/E]^2 \end{aligned} \right\}. \quad (8)$$

The two real roots of equation (6) determine the common boundary as

$$\zeta^\pm = -\frac{A_2}{2A_1} \pm \left[\left(\frac{A_2}{2A_1} \right)^2 - \frac{A_3}{A_1} \right]^{1/2}, \quad (9)$$

and hence

$$(N_r, N_\theta, M_r, M_\theta) = \int_{-h/2}^{\zeta^-} \mathbf{T}^p dz + \int_{\zeta^-}^{\zeta^+} \mathbf{T}^e dz + \int_{\zeta^+}^{h/2} \mathbf{T}^p dz; \tag{10}$$

this holds for all regions of the plate as long as one replaces ζ^- and ζ^+ with $(-h/2)$ and $(h/2)$, respectively, for region I, replaces ζ^+ with $(h/2)$ for region II, replaces ζ^- with $(-h/2)$ for region IV and deletes superfluous terms at the same time. In equation (10),

$$\mathbf{T} = (\sigma_r, \sigma_\theta, z\sigma_r, z\sigma_\theta). \tag{11}$$

Usually, numerical integration is necessary for the calculation of equation (10), owing to the variation of E_s with spatial coordinates. However, if the material of the circular sheet is an elastic-perfectly plastic one, explicit expressions for N_r , etc., can be derived [9] as

$$\left. \begin{aligned} N_r &= \frac{E}{1-\nu^2} \{A_r^* h + A_r^{**} [h(\zeta^- + \zeta^+) - (\zeta^{+2} - \zeta^{-2})]\} \\ N_\theta &= \frac{E}{1-\nu^2} \{A_\theta^* h + A_\theta^{**} [h(\zeta^- + \zeta^+) - (\zeta^{+2} - \zeta^{-2})]\} \\ M_r &= \frac{EB_r}{1-\nu^2} \left[\frac{h^2}{4} (\zeta^+ - \zeta^-) - (\zeta^{+3} - \zeta^{-3})/3 \right] \\ M_\theta &= \frac{EB_\theta}{1-\nu^2} \left[\frac{h^2}{4} (\zeta^+ - \zeta^-) - (\zeta^{+3} - \zeta^{-3})/3 \right] \end{aligned} \right\}, \tag{12}$$

where

$$\left. \begin{aligned} A_r^* &= \varepsilon_r^0 + \nu \varepsilon_\theta^0, & A_r^{**} &= \frac{1}{2}(K_r + \nu K_\theta) \\ A_\theta^* &= \varepsilon_\theta^0 + \nu \varepsilon_r^0, & A_\theta^{**} &= \frac{1}{2}(K_\theta + \nu K_r) \\ B_r &= A_r^{**}, & B_\theta &= A_\theta^{**} \end{aligned} \right\}. \tag{13}$$

Numerical results can be obtained easily with the aid of the maDR method [1, 10]. For a general material, one should use expression (10), while for an elastic-perfectly plastic material, the application of equation (12) is convenient.

2.2. Unloading process

Assuming that the state of a point in the plate is at O^* on the loading stress-strain curve at the end of the loading process (cf. Fig. 3), one can establish an unloading coordinate system

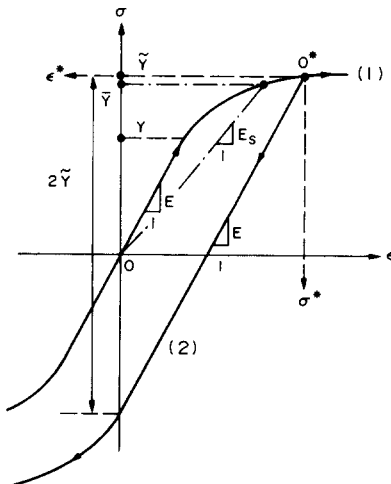


FIG. 3. Coordinate systems of loading and unloading.

(UCS) $\varepsilon^0 \sigma^*$ whose axis directions are opposite to those of the corresponding loading ones. Letting \bar{w} be the deflection in the loading coordinate system (LCS), w^* be that in UCS and \tilde{w} be the deflection at the end of loading process in the LCS, introducing similar notation for stresses and strains, etc., and considering the directions of the principal axes of stresses and strains in LCS and UCS, one obtains

$$\begin{aligned}\bar{w} &= \tilde{w} + w^*, & \bar{u} &= \tilde{u} + u^*, \\ \bar{\varepsilon}_r &= \tilde{\varepsilon}_r + \varepsilon_r^*, & \bar{\varepsilon}_\theta &= \tilde{\varepsilon}_\theta + \varepsilon_\theta^*,\end{aligned}\quad (14)$$

etc.

Obviously, \bar{w} and \tilde{w} , etc., satisfy the equations in the loading process—see equations (1) and (2). The substitution of equation (14) into these equations therefore yields the equations in the UCS as follows:

$$\left. \begin{aligned}\frac{\partial N_r^*}{\partial r} + \frac{1}{r}(N_r^* - N_\theta^*) &= 0, \\ \frac{\partial^2 M_r^*}{\partial r^2} + \frac{1}{r} \left(2 \frac{\partial M_r^*}{\partial r} - \frac{\partial M_\theta^*}{\partial r} \right) + \tilde{N}_r \frac{\partial^2 w^*}{\partial r^2} + N_r^* \left(\frac{\partial^2 \tilde{w}}{\partial r^2} + \frac{\partial^2 w^*}{\partial r^2} \right) \\ + \frac{1}{r} \left[\tilde{N}_\theta \frac{w^*}{r^2} + N_\theta^* \left(\frac{\partial \tilde{w}}{\partial r} + \frac{\partial w^*}{\partial r} \right) \right] + q^* &= 0\end{aligned}\right\} \quad (15)$$

and

$$\left. \begin{aligned}\varepsilon_r^{0*} &= \frac{\partial u^*}{\partial r} + \frac{\partial \tilde{w}}{\partial r} + \frac{\partial w^*}{\partial r} + \frac{1}{2} \left(\frac{\partial w^*}{\partial r} \right)^2 \\ \varepsilon_\theta^{0*} &= \frac{u^*}{r} \\ K_r^* &= -\frac{\partial^2 w^*}{\partial r^2} \\ K_\theta^* &= -\frac{1}{r} \frac{\partial w^*}{\partial r}\end{aligned}\right\} \quad (16)$$

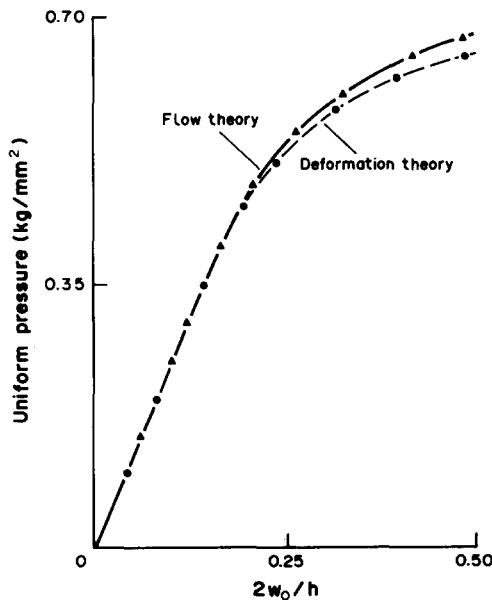


FIG. 4. Variation curve of lateral pressure—central deflection of 250-10.0-UC.

The form of the constitutive equations in the UCS is the same as that in equation (10) for a general material or is the same as that in equation (12) for an elastic–perfectly plastic one. However, it should be noted that in the UCS, the Mises condition becomes

$$\sigma_{\theta}^{*2} + \sigma_r^{*2} - \sigma_{\theta}^* \sigma_r^* = 4 \tilde{Y}^2, \tag{17}$$

where \tilde{Y} is the stress at 0*; see Fig. 3.

3. DISCUSSION AND CONCLUSIONS

Solutions from J_2 deformation theory obtained by solving the above non-linear equations with the aid of the maDR method [1, 10] have been obtained and compared with corresponding solutions from the simple J_2 flow theory, which were found to be in good agreement with experimental results [1, 2]. It follows from Figs 4–6 that solutions for 250-10.0-UC from J_2 deformation theory accord well with those from the simple J_2 flow theory, where the loading paths are closer to proportional loading. However, for 250-10.0-C70 (see Figs 5 and 7), significant discrepancies exist between the solutions from the two

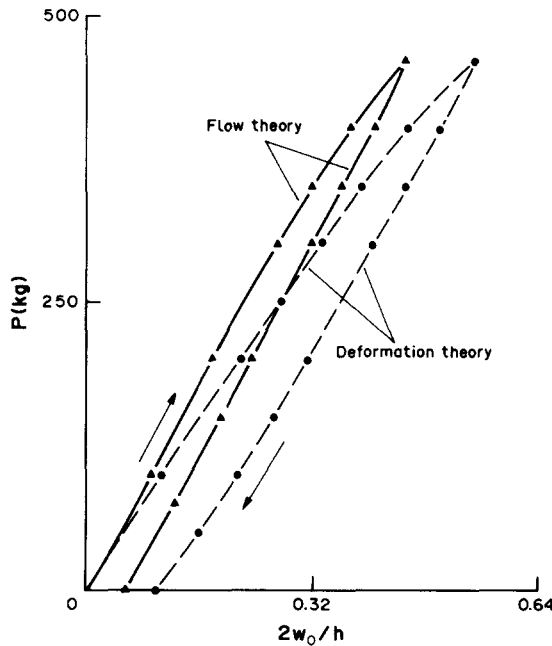


FIG. 5. Variation curve of lateral load–central deflection of 250-10.0-C70.

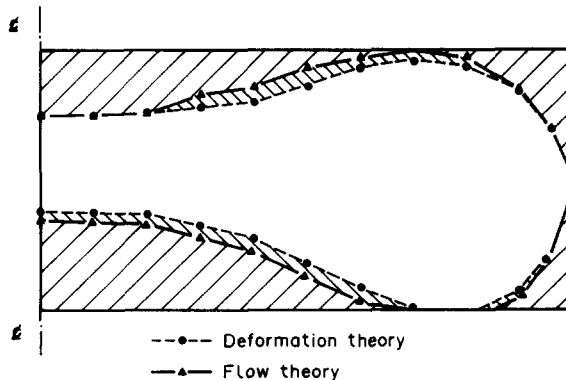


FIG. 6. Comparison of plastic regions in radial section of 250-10.0-UC with lateral pressure 0.653 kg mm^{-2} .

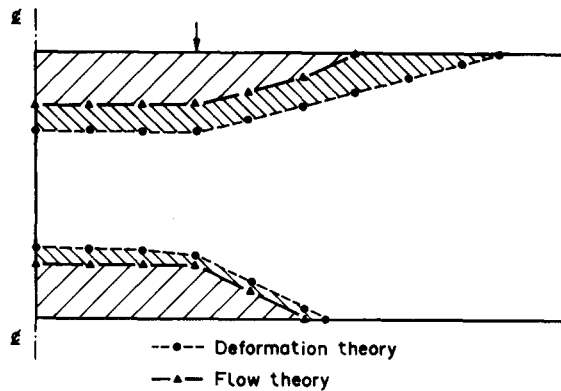


FIG. 7. Comparison of plastic regions in radial section of 250-10.0-C70 with lateral load 450 kg.

theories, although there is no local unloading during loading and the loading paths are not far from the proportional one. In addition, for the plates of 150-2.0-C45 and 150-2.0-C65, solutions from the deformation theory are so much at variance with those of flow theory or that rendered by experimental methods that they are useless for engineering purposes. It is therefore obvious that although they may be used for a problem with no local unloading during the loading process, one should be careful in applying deformation theory if the loading paths are not closer to those of proportional loading. It follows that the applicable potential of deformation theory is not as optimistic as has been imagined previously, at least in the case of sheet stamping.

It should, however, be noted that the extent of any deviation from the proportional loading path is very much related to the distribution and form of the lateral load. For a problem with a lateral load distributed gently, the load paths will be close to the proportional paths, whilst for those which vary sharply, the former will deviate far from the latter. Hence, one may expect that deformation theories will lead to reasonable and useful results—for instance, in hydroforming problems.

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