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An analytical solution to springback of sheet metals stamped by a rigid punch and an elastic die

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Industrial Summary

Springback is one of the fundamental factors affecting the quality of a stamped component. It has been recognised that springback can be controlled more easily when using an elastic die in a stamping process. However, the deformation of the die introduces more complex mechanisms of springback and an in-depth understanding of such processes is still lacking. This paper developed an analytical solution to the springback of sheet metals stamped by a rigid punch and an elastic die under plane-stress deformation. The stamping process was modelled as a three-body contact problem. The sheet workpiece was divided into three parts according to its contact with the punch and die. The workpiece material was considered to be elastic/perfectly plastic. Based on the solution obtained, the effects of the most important processing factors, namely the friction between the workpiece and die and the elastic properties of the die, were examined. The solution shows that a no-springback stamping process can be achieved by a comprehensive selection of processing parameters.

Nomenclature

c	half length of the contact zone between the punch and sheet, Fig. 3(a)
c'	half length of the contact zone between the sheet and die, Fig. 3(a)
C_1, C_2	constants to be determined by boundary conditions, see Eqs. (7) and (8)
E	Young's modulus
m	dimensionless bending moment, defined by Eq. (4)
n	dimensionless shear force, defined by Eq. (4)
q	contact stress
R	radius of punch
t	dimensionless membrane force, defined by Eq. (4)
w	deflection of the sheet (in y-direction), Fig. 3(a)
w'	deflection of the cantilever beam (vertical to z-axis), Fig. 3(c)
x, y	global Cartesian coordinates, defined by Eq. (4), see also Fig.3(a)
z	local axial coordinate attached to the cantilever beam with its origin at end A, Fig. 3(c)
θ	polar coordinate variable, Fig. 3(a)
ξ	springback ratio, defined by Eq. (14)
φ	included angle of the tangent of the deformed sheet surface at θ with the positive direction of x-axis, Fig. 3(a)
ψ	dimensionless curvature, defined by Eq. (4)

Subscript

d	die
e	elastic
n	normal direction
p	punch
t	tangential direction

1. Introduction

Because of the practical importance of sheet metal stamping using deformable forming tools [1], theoretical and experimental studies on sheet deformation have been carried out extensively [1-4]. However, owing to the complexity of the deformation mechanisms involved, investigations into the stamping processes with deformable tools are difficult. Consequently, there are two ways that have been used to gain a deeper understanding of the stamping processes. One is to develop reliable methods and models to reveal the qualitative behaviour, and the other is to carry out parametric studies to generate quantitative guidelines for the design of practical operations.

This paper aims to develop an analytical approach to study the springback of sheet metals stamped by a rigid punch and an elastic die. Based on this, the dependence of springback on the main stamping parameters are discussed in detail.

2. Modelling

Consider an elastic/perfectly plastic metal sheet (thickness h, yield stress σ_Y, Young's modulus E_s) on an elastic die (Young's modulus E_d) stamped by a rigid punch of circular cylinder of radius R, see Fig. 1. The stamping is a three-body contact problem but is under plane-stress deformation. Because of the large friction between the surfaces of the sheet and die, the large deflection theory of thin strip bending must be employed.

Assume that a plane section of the sheet remains plane throughout the stamping process. Then strain across the plate thickness is always linear. Correspondingly, there are three possible stress states at any cross-section of the sheet, i.e., pure elastic (S1), single-side plastic (S2) and double-side plastic (S3), depending on the different combinations of the bending moment M and membrane force T at that cross-section, see Fig. 2.

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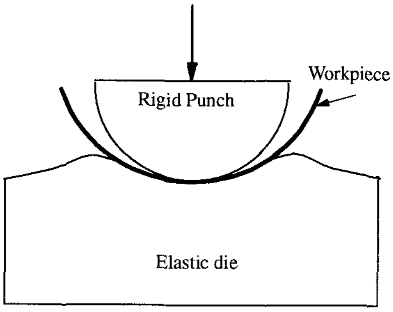

Fig. 1. The schematic of stamping with an elastic die

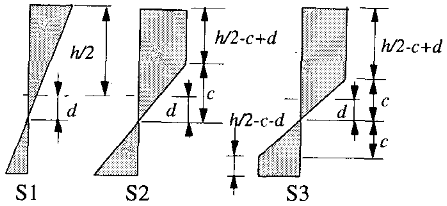

Fig. 2. Possible stress states in the sheet workpiece

The deformation of the sheet is symmetrical to its central section (Fig. 1). Thus only half of the sheet needs to be studied. The contact zones and interface stresses between the punch and sheet and between the sheet and die are unknown in advance, which are functions of the punch stroke in the stamping process. In order to analyse the problem properly, similar to the analysis of stamping a sheet elastically into an elastic foundation [5], we can divide the sheet into three parts according to its contact with the punch and die (Fig. 3(a)): (1) the central part A'A, where the sheet is in perfect contact with both the punch and die surfaces, (2) the transition part AC, where the strip is in perfect contact with the die but has no contact with the punch, and (3) the free part CD, where no contact takes place with either the punch or die .

The curvature of the central part is a known function which is identical to that of the punch surface, i.e., the curvature of the circle with radius R. The contact stresses on the sheet of this part are the normal and tangential contact stresses between the sheet and punch, q_{np} and q_{tp}, and the normal and tangential ones between the sheet and elastic die, q_{nd} and q_{td}. Usually, q_{tp} is much smaller than q_{td}. Thus, for simplicity, we will ignore q_{tp} in our analysis, see Fig. 3(b).

The transition part can be modelled as a cantilever beam subjected to both normal and tangential stresses, q_{nd} and q_{td}, due to the interaction between the sheet and die, see Fig. 3(c). The end A of this part is the contact-off point between the punch and sheet, and end C is the contact-off point between the sheet and elastic die. The boundary conditions of the beam must guarantee the continuity of stresses and deformation across the two ends. This requires that the bending moment, membrane and shear forces are zero at C, and that the deflection and its slope, bending moment and membrane force are equal to those of the central part at A.

The free part of the sheet, CD, does not deform during stamping. Its displacement relies on the deflection and deflection slope at end C. This part has no contribution to the springback of the sheet, and therefore will be ignored in the following analysis.

To simplify the calculation of the contact stresses, we assume that the normal reaction of the elastic die follows Winkler's hypotheses, and that the tangential contact stress between the sheet and die is proportional to the normal stress, i.e. $q_{td} = \mu q_{nd}$, where μ is the friction coefficient.

In the next section, we will first obtain the analytical solutions to parts A'A and AC, and then calculate stresses using a numerical scheme in conjunction with the compatibility conditions of deformation between the two parts.

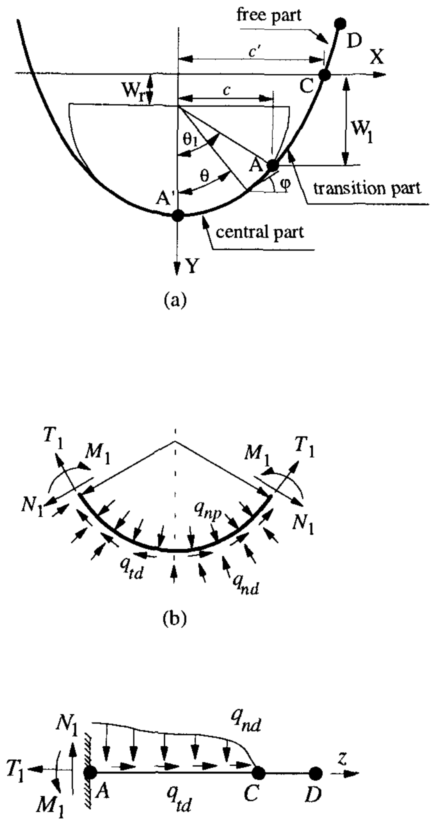

Fig. 3. The mechanics modelling

3. Solution

3.1. Basic equations

Based on the above mechanics analysis and the theory of thin strip with large deflection, the equilibrium, geometric, and compatibility equations can be written in the following forms:

$$\begin{cases} \dfrac{dt}{d\theta} + n + \eta_1 q_t = 0, \\ \dfrac{dn}{d\theta} - t + \eta_1 q_n = 0, \\ \dfrac{dm}{d\theta} - 6\eta_1 n = 0, \end{cases} \tag{1}$$

$$\frac{dx}{ds} = \cos\varphi, \quad \frac{dy}{ds} = \sin\varphi, \quad \frac{d\varphi}{ds} = \eta_3\psi, \tag{2}$$

$$\begin{cases} w(\theta_1^-) = w(\theta_1^+), \quad \varphi(\theta_1^-) = \varphi(\theta_1^+), \\ m(\theta_1^-) = m(\theta_1^+), \quad t(\theta_1^-) = t(\theta_1^+), \end{cases} \tag{3}$$

where

$$\begin{cases} m = \dfrac{M}{M_e}, \; n = \dfrac{N}{N_e}, \; t = \dfrac{T}{T_e}, \; \psi = \dfrac{\kappa}{\kappa_e}, \; q = \dfrac{Q}{\sigma_y}, \\ x = \dfrac{X}{R}, \; y = \dfrac{Y}{R}, \; ds = \dfrac{dS}{R}, \; w = \dfrac{W}{R}, \eta_1 = \dfrac{R}{h}, \eta_3 = \dfrac{R}{\kappa_e}, \end{cases} \tag{4}$$

in which $M_e = \sigma_Y h^2/6$ is the maximum elastic bending moment, $N_e = \sigma_Y h$ the maximum elastic shear force, $T_e = \sigma_Y h$ the maximum elastic membrane force, Q the contact stress, W the deflection of the sheet, κ the curvature of the sheet, and $\kappa_e = 2\sigma_Y/hE_s$ is the maximum elastic curvature of the sheet. Other notations can be found in the list of nomenclature. According to the deformation assumption of the die material and the geometrical relations shown in Fig. 3, the contact stresses between the sheet and die can be expressed in terms of the deflection of the sheet, w_1, at the contact-off section A, that is,

$$\begin{cases} q_{nd} = \eta_2(w_1 - \cos\theta_1 + \cos\theta), \\ q_{td} = \mu\eta_2(w_1 - \cos\theta_1 + \cos\theta), \end{cases} \tag{5}$$

where $\eta_2 = E_d/(2\sigma_Y)$ is a non-dimensional constant.

3.2. Solution to the central part

In the central part A'A, the curvature of the sheet is known, because the sheet is in complete contact with the punch. Thus the solution to the above basic equations can be obtained as follows:

(a) When $0 \le m \le 1 - t$, the stress state is pure elastic,

$$\begin{cases} m = \psi, \\ n = 0, \\ t = t_0 - \eta_1\eta_2\mu\left[(w_1 - \cos\theta_1)\theta + \sin\theta\right], \end{cases} \tag{6}$$

where $t_0 = t|_{\theta=0}$ and $w_1 = w|_{\theta=\theta 1}$.

(b) When $1 - t \le m \le 1 + t - 2t^2$, the stress state is single-side plastic. This gives rise to

$$\begin{cases} (2\eta_1 - 1)t - \dfrac{2\sqrt{(1-t)^3}}{3\sqrt{\psi}} = C_1 \\ \quad - 2\eta_1^2\eta_2\mu\left[(w_1 - \cos\theta_1)\theta + \sin\theta\right], \end{cases} \tag{7a}$$

$$\begin{cases} m = (1-t)\left(3 - 2\dfrac{\sqrt{1-t}}{\sqrt{\psi}}\right), \\ n = \dfrac{\eta_1 q_t\left(\sqrt{\psi} - \sqrt{1-t}\right)}{\sqrt{1-t} + \sqrt{\psi}(2\eta_1 - 1)}, \end{cases} \tag{7b, c}$$

(c) When $1 + t - 2t^2 \le m \le 1.5\,(1 - t^2)$, the stress state is double-side plastic, which leads to

$$\begin{cases} 4\eta_1 t - t^2 = C_2 - 4\eta_1^2\eta_2\mu\left[(w_1 - \cos\theta_1)\theta + \sin\theta\right], \\ m = \dfrac{1}{2}\left[3(1-t^2) - \dfrac{1}{\psi^2}\right], \\ n = \dfrac{\eta_1 q_t t}{2\eta_1 - t}. \end{cases} \tag{8}$$

With the internal forces m and t determined above, the contact stress q_{np} can be obtained by

$$q_{np} = \left(t - \frac{dn}{d\theta}\right)\frac{1}{\eta_1} + q_{nd} \tag{9}$$

The detailed derivation of the above solutions can be found in the appendix of this paper.

3.3. Solution to Part AC

The deflection of part AC can be expressed as

$$w' = \int_z^{L'} \frac{\sin\varphi}{\cos\varphi}\, d\xi, \tag{10}$$

where L' is the length of part AC. To calculate w' using Eq. (10), it is necessary to find the relationship between the axial coordinate z and the slope of the deformed plate surface, that is,

$$\frac{d\varphi}{dz} = \frac{\eta_3\psi}{\cos\varphi}, \qquad \sin\varphi = \eta_3\int_z^{L'} \psi\, d\xi. \tag{11}$$

$$\psi = \begin{cases} m, & 0 \le m \le 1-t \\ \dfrac{4(1-t)^3}{\{3(1-t)-m\}^2}, & 1-t \le m \le 1+t-2t^2 \\ \{3(1-t^2)-2m\}^{-1/2} & 1+t-2t^2 \le m \le 1.5(1-t^2) \end{cases} \tag{12}$$

where

$$\begin{cases} m = \eta_1^2\eta_2(L')^2\displaystyle\int_z^1\int_\xi^1\left[w_1\left(1 - \frac{\zeta}{L}\right) - w'\cos\theta_1\right]d\zeta\, d\xi, \\ t = \eta_1\eta_2 L'\displaystyle\int_z^1\left[w_1\left(1 - \frac{\zeta}{L}\right) - w'\cos\theta_1\right]d\zeta. \end{cases} \tag{13}$$

Equations (10) to (13) are the relations for calculating the deflection of part AC. Because of the strong non-linearity, they must be solved by an iteration scheme.

3.4. Springback

Assume that the unloading process is pure elastic. Then, the springback ratio ξ of the sheet at any section is given by [4]

$$\xi = 1 - \frac{m}{\psi} \tag{14}$$

when the punch is removed. ξ varies between 0 and 1, with $\xi = 1$ indicating no-springback and $\xi = 0$ a full springback corresponding to a pure elastic stamping. Hence, a larger ξ represents a smaller springback and *vice versa*. With m and ψ determined by solutions (6) to (13), ξ can be obtained easily.

3.5. Iteration technique

Quantities w_1 and t_0 in the solutions of parts A'A and AC must be determined by the deformation compatibility between the two parts specified by Eq. (3). For convenience, the iteration can start with ignoring the deformation of part AC. The procedure is as follows.

(*i*) Give contact-off angle θ_1.

(*ii*) Calculate the internal force of parts A'A and AC ignoring the deformation of part AC and determine $w_1^{(0)}$, $t_0^{(0)}$ by compatibility conditions.

(*iii*) Calculate the deflection of part AC, using

$$(w')^{(0)} = L\int_z^1 \tan\varphi^{(0)}\, d\xi, \quad \sin\varphi^{(0)} = L\eta_1\eta_2\int_z^1 \psi^{(0)}\, d\zeta,$$

$$\psi^{(0)} = \begin{cases} m^{(0)}, & 0 \le m \le 1-t, \\ \dfrac{4\left(1-t^{(0)}\right)^3}{\left\{3\left(1-t^{(0)}\right)-m^{(0)}\right\}^2}, & 1-t \le m \le 1+t-2t^2 \\ \left\{3\left[1-\left(t^{(0)}\right)^2\right]-2m^{(0)}\right\}^{-1/2} & 1+t-2t^2 \le m \le 1.5\left(1-t^2\right) \end{cases}$$

where $L = L'^{(0)}$ is the initial length of part AC.

(*iv*) Revise $w_1^{(i)}$, $t_0^{(i)}$ and the contact stresses of parts A'A and AC.

(*v*) Determine instant contact-off point C where $q_{nc}^{(i)} = 0$, and then determine $L'^{(i)}$ by

$$\begin{cases} q_{nc}^{(i)} = \eta_2 w_1^{(i)}\left(1-\dfrac{z}{L}\right) - (w')^{(i-1)}\eta_2\sin\theta_1, \\ q_{tc}^{(i)} = q_{nc}^{(i)}\mu. \quad 0 \le z \le L'^{(i)} \end{cases}$$

(*vi*) Calculate the deflection of part AC by

$$(w')^{(i)} = L'^{(i)}\int_z^1 \tan\varphi^{(i)} d\xi, \quad \sin\varphi^{(i)} = L'^{(i)}\eta_1\eta_2\int_z^1 \psi^{(i)}\, d\zeta,$$

$$\psi^{(i)} = \begin{cases} m^{(i)}, & 0 \le m \le 1-t, \\ \dfrac{4(1-t^{(i)})^3}{\left\{3\left[1-t^{(i)}\right]-m^{(i)}\right\}^2}, & 1-t \le m \le 1+t-2t^2 \\ \left\{3\left[1-\left(t^{(i)}\right)^2\right]-2m^{(i)}\right\}^{-1/2} & 1+t-2t^2 \le m \le 1.5(1-t^2) \end{cases}$$

where

$$m^{(i)} = \eta_1^2\eta_2(L'^{(i)})^2 \int_z^1\int_\xi^1 q_{nc}^{(i)}\, d\zeta d\xi, \quad t^{(i)} = \eta_1\eta_2 L'^{(i)} \int_z^1 q_{tc}^{(i)}\, d\zeta.$$

An internal iteration must be conducted inside this step until all the compatibility conditions are satisfied.

(*vii*) Check the convergence criterion

$$\left| \frac{w_1^{(i)} - w_1^{(i-1)}}{w_1^{(i)}} \right| = \varepsilon.$$

If it is satisfied, do the next stop; otherwise, return to step (*iv*), where ε is a small positive number.

(*iix*) Calculate the springback ratio by Eq. (14).

4. Numerical examples

Figure 4 shows the variation of the springback ratio with the change of the contact angle θ_1 when geometrical and material parameters are given. The increase of θ_1 raises the springback ratio. This is because a larger θ_1 indicates a greater depth of indentation and thus more significant plastic deformation.

When θ_1 is small, say less than 18°, the bending moment in the central part of the sheet is uniform (Fig. 5(a)), indicating a pure bending in this part. However, when θ_1 reaches to a certain value, 67.5° for instance, the bending moment is no longer uniform (Fig. 5(b)), because membrane force t increases considerably (Fig. 5(c)). It is evident that t plays an important role in the variation of ξ. It is interesting to note that the variation tendency of ξ is very similar to that of t.

The friction between the sheet and die affects greatly the behaviour of ξ, see Fig. 6. It is obvious that a die possessing a larger coefficient of friction is desirable in terms of a higher springback ratio. We can see clearly that when μ is beyond 0.150, most part of the sheet will not have springback because of $\xi = 1$. An interpretation of the above effect is that the friction increases the membrane force. In addition, a larger friction helps to prevent from relative sliding between the sheet and die surfaces and thus avoid scratching the workpiece surface.

Increasing the value of η_2, while keeping σ_Y a constant, means to increase the elastic modulus of the die. Figure 7 demonstrates the effect of the elastic modulus of die on the springback ratio. Similar to the friction effect, the use of a die with a sufficiently larger elastic modulus will also raise ξ to one and therefore give rise to a no-springback stamping. However, this will, at the same time, increase the total stamping load that is generally not favourable in practical operations. Thus the selection of the die material should be based on a comprehensive consideration of both E_d and μ.

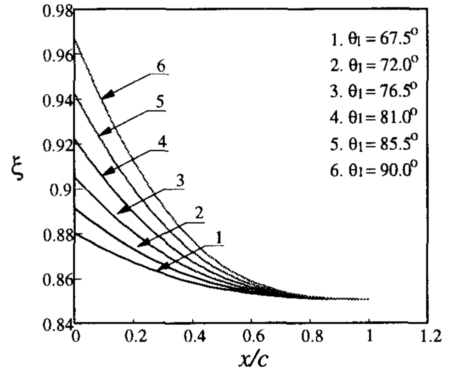

Fig. 4. Variation of springback ratio (μ = 0.05, η_1 = 50, h = 1.0, E_s/E_d = 2000, E_s/σ_Y = 1000, σ_Y = 210 MPa)

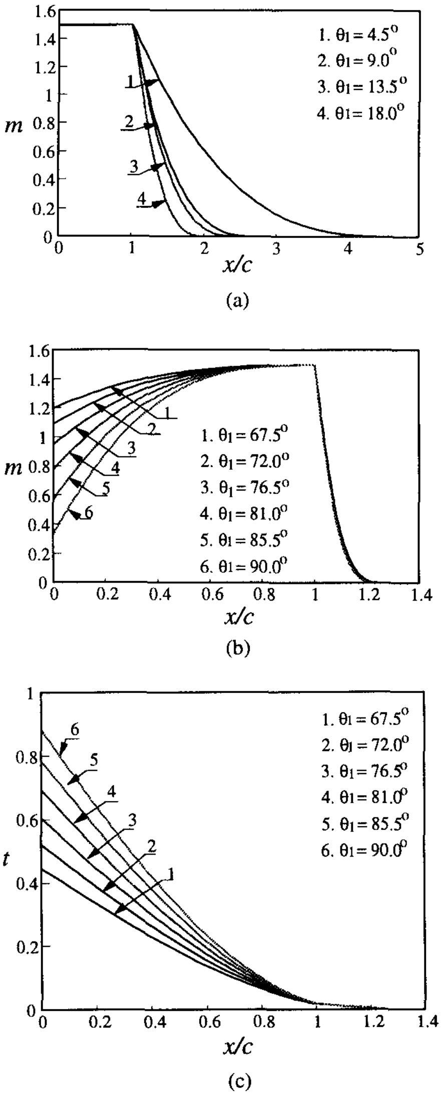

Fig. 5. Distribution of internal forces (μ = 0.05, η_1 = 50, h = 1.0, E_s/E_d = 2000, E_s/σ_Y = 1000, σ_Y = 210 MPa)

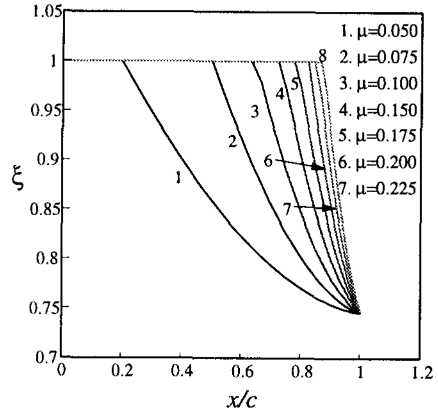

Fig. 6. Effect of friction on the springback ratio (θ_1 = 90°, η_1 = 50, h = 1.0, E_s/E_d = 2000, E_s/σ_Y = 1000, σ_Y = 210 MPa)

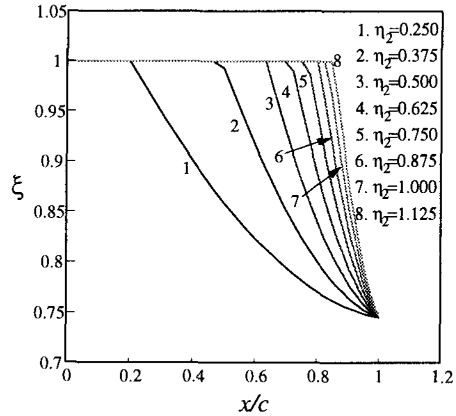

Fig. 7. Effect of elastic modulus of die on the springback ratio (μ = 0.05, θ_1 = 90°, η_1 = 50, h = 1.0, E_s/σ_Y = 1000, σ_Y = 210 MPa)

5. Conclusions

(1) An analytical solution to the springback of sheet metals stamped by a rigid punch and an elastic die has been generated. The method developed can be extended to more complex stamping problems.

(2) A no-springback stamping process, which is favourable in manufacturing practice, can be obtained when using elastic dies. An optimal selection of a die material should be based on a comprehensive consideration of both the coefficient of friction and the elastic modulus of the die.

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Appendix: Some details of solution to part A'A

(i) In the single-side plastic state,

$$\begin{cases} m = (1-t)\left[3 - 2\dfrac{\sqrt{1-t}}{\sqrt{\psi}}\right], \\ n = \dfrac{1}{6\eta_1}\dfrac{dm}{d\theta}, \\ q_t = -\dfrac{1}{2\eta_1^2}\left[\dfrac{\sqrt{1-t}}{\sqrt{\psi}} + (2\eta_1 - 1)\right]\dfrac{dt}{d\theta}, \\ \dfrac{dn}{d\theta} = t - \eta_1 q_n, \end{cases}$$

Clearly, m and n vary even ψ is a constant. The solution is therefore

$$\begin{cases} (2\eta_1 - 1)t - \dfrac{2\sqrt{(1-t)^3}}{3\sqrt{\psi}} = C_1 - 2\eta_1^2\eta_2\mu\left[(w_1 - \cos\theta_1)\theta + \sin\theta\right], \\ n = \dfrac{1}{2\eta_1}\left[\dfrac{\sqrt{1-t}}{\sqrt{\psi}} - 1\right]\dfrac{dt}{d\theta}, \\ \dfrac{dn}{d\theta} = \dfrac{1}{2\eta_1}\left[\left(\dfrac{\sqrt{1-t}}{\sqrt{\psi}} - 1\right)\dfrac{d^2t}{d\theta^2} - \dfrac{1}{2\sqrt{\psi(1-t)}}\left(\dfrac{dt}{d\theta}\right)^2\right], \end{cases}$$

where

$$\begin{cases} \dfrac{dt}{d\theta} = \dfrac{-2\eta_1^2 q_t\sqrt{\psi}}{\left[\sqrt{1-t} + \sqrt{\psi}(2\eta_1 - 1)\right]}, \\ \dfrac{d^2t}{d\theta^2} = \dfrac{-2\eta_1^2\sqrt{\psi}}{\left[\sqrt{1-t} + \sqrt{\psi}(2\eta_1 - 1)\right]}\dfrac{dq_t}{d\theta} \\ \qquad - \dfrac{2\eta_1^2 q_t\sqrt{\psi}}{\left[\sqrt{1-t} + \sqrt{\psi}(2\eta_1 - 1)\right]^2\sqrt{1-t}}\dfrac{dt}{d\theta}. \end{cases}$$

(ii) When the stress state is double-side plastic, we have

$$\begin{cases} m = \dfrac{1}{2}\left[3(1-t^2) - \dfrac{1}{\psi^2}\right], \\ q_t = -\dfrac{1}{\eta_1}\left[1 - \dfrac{t}{2\eta_1}\right]\dfrac{dt}{d\theta}, \\ \dfrac{dn}{d\theta} = t - \eta_1 q_n, \\ \dfrac{dm}{d\theta} = 6\eta_1 n. \end{cases}$$

This gives rise to

$$\begin{cases} 4\eta_1 t - t^2 = C_2 - 4\eta_1^2\eta_2\mu\left[(w_1 - \cos\theta_1)\theta + \sin\theta\right], \\ n = -\dfrac{t}{2\eta_1}\dfrac{dt}{d\theta}, \\ \dfrac{dn}{d\theta} = -\dfrac{1}{2\eta_1}\left[t\dfrac{d^2t}{d\theta^2} + \left(\dfrac{dt}{d\theta}\right)^2\right], \\ \dfrac{dt}{d\theta} = \dfrac{-2\eta_1^2 q_t}{2\eta_1 - t}, \\ \dfrac{d^2t}{d\theta^2} = \dfrac{-2\eta_1^2}{(2\eta_1 - t)}\dfrac{dq_t}{d\theta} + \dfrac{2\eta_1^2 q_t}{(2\eta_1 - t)^2}\dfrac{dt}{d\theta}. \end{cases}$$