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Surface roughness prediction of ground components using a fuzzy logic approach

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Abstract

In this paper a total of 16 variables, which are most influential on surface roughness in grinding, are considered. The variables are classified into three groups depending on their significance and effect on surface roughness. A three-layer fuzzy model is used to correlate these variables to surface roughness using the fuzzy rules generated based on experimental observations and recommendations from wheel manufacturers. Membership functions, fuzzy rule bases, and a worked example are presented in detail to demonstrate the strength of fuzzy logic in modeling such a complex system in an efficient manner. © 1999 Elsevier Science S.A. All rights reserved.

Keywords: Grinding; Surface roughness; Fuzzy logic; Membership functions

1. Introduction

Grinding is a surface finishing process, and surface roughness is one of the most important factors in assessing the quality of a ground component. However, there is no comprehensive model that can predict roughness over a wide range of operating conditions; and after many decades of research, this is an area that relies on the experience and skills of machine operators. The difficulty stems from the fact that many variables are affecting the process. These include: work material properties, grinding wheel composition, dressing conditions, operation parameters, coolant application and properties, and machine vibration. Many of these variables are non-linear, interdependent, or difficult to quantify with crisp numeric precision. Therefore, physical models are not feasible, and experimental investigations can be very exhaustive and with limited applicability.

An added difficulty is the fact that there are many ways to measure and characterize a surface [1]. There is no single number that can be used to fully describe surface topography. Therefore, for the present work, it is assumed that surface roughness is characterized by the center-line-average (CLA) of a line profile sampled using a stylus moving parallel to the grinding wheel's axis, notwithstanding the resulting ambiguity and loss of information.

Theoretical models for predicting 'ideal' surface roughness along the longitudinal and transverse directions have been reviewed by Malkin [2]. These ideal models assume an analogy with the milling process and show that surface roughness depends on table speed, wheel speed, grit spacing, grit radius of curvature, wheel diameter, and cross-feed overlay; and is independent of depth of cut or coolant application. However, these models predict erroneous values that are one to three orders of magnitude smaller than the 'real' surface roughness. Tawfik [3] treated surface grinding as a mixture of micro-milling and wear processes, and showed that roughness also depends the on depth of cut and work material hardness. The model is accurate but requires some experimental calibration.

Lindsay [4] showed experimentally that roughness depends on the material removal rate, the undeformed chip thickness, and the diamond dressing in-feed and lead. Yet, Malkin [2] pointed out that dressing affects

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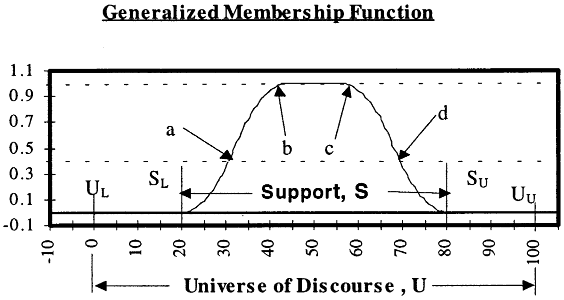

Fig. 1. Definition of a membership function, Eqs. (1)–(3).

roughness only when a fresh wheel or continuous dressing is applied. Once a wheel reaches 'steady state' surface topography, work roughness becomes independent of the dressing conditions. Further, Kannappan and Malkin [5] showed experimentally that roughness depends on grit size, and on wheel grade to a lesser extent. One of the most comprehensive experimental investigations is that by Farmer, Brecker, and Shaw [6], in which they demonstrated the effect of more than eight variables on surface roughness. They used their results to construct a factorial design which is used to make 'relative' predictions of surface roughness.

Tonshoff et al. [7] reviewed all theoretical and experimental models for surface roughness dated from 1952 to 1992. They showed that various models considered many grinding variables, and proposed some basic general models that cover many of the variables. However, they noticed that no single model (theoretical or empirical) takes the effect of coolant into account. Experience shows that coolant application can play a significant role on surface roughness [8] and should not be ignored. A coolant not only acts as a lubricant and smoothens the rubbing of the wheel on the surface, but also as a surface cleaner. By flushing grinding chips away from the grinding zone, it ensures finer penetration by grits, and prevents re-welding of chips to the ground surface, thus improving the surface finish. Further, from a contact mechanics point of view, coolant affects the mechanical properties of a grinding wheel and in turn influences the surface roughness [9,10].

Given the above complexity of physical phenomena and confusion in experimental results, Zhang et al. [9]

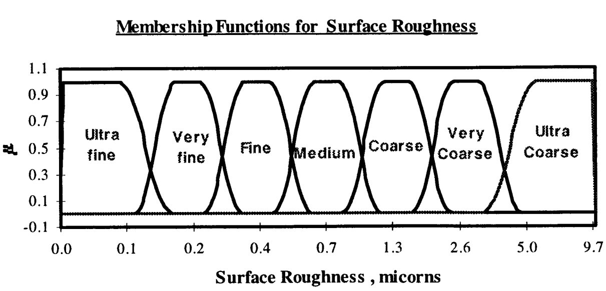

Fig. 2. Fuzzy surface roughness.

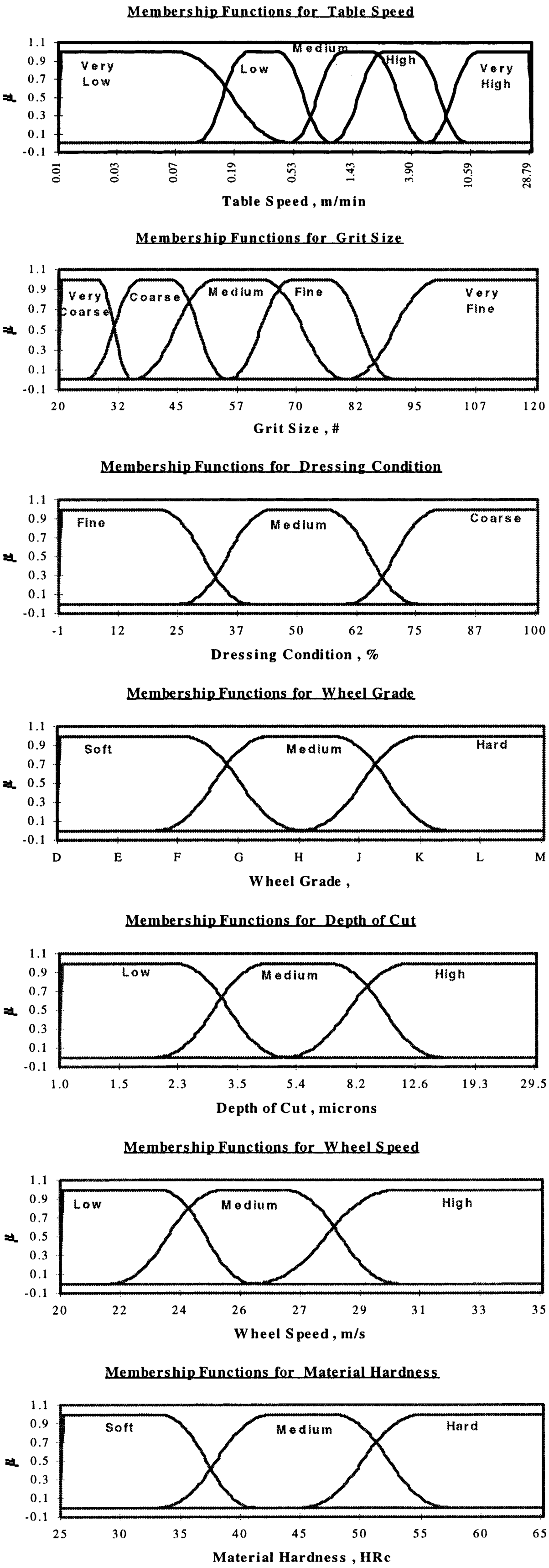

Fig. 3. Membership functions for primary variables.

pointed out that dimensional analysis can lead to more comprehensive models. Shaw [11] suggested the impracticality of any absolute modeling of surface roughness and proposed a relative approach.

In relative analysis, investigation of dimensionless parameters and factorial design are used to construct relative models. These are models that predict the change in surface roughness at some grinding state compared with the known surface roughness at another grinding state. An inherent problem with this approach is that it assumes linear behavior between the two states, and therefore, cannot be applied over a wide range of grinding conditions.

This paper proposes a fuzzy model for surface roughness estimation that takes many variables into account and covers a wide range of grinding conditions. Moreover, the model can utilize the overwhelming experience gained over decades by experienced grinding technicians, which cannot be done by any of the existing models. This work is encouraged by the earlier success of fuzzy logic in modeling the residual stresses induced by grinding [12] and in grinding wheel selection [13].

2. The fuzzy model for surface roughness

2.1. Fuzzy logic principles

Fuzzy logic is concerned with the continuous transition from truth to falsity states [14], as opposed to the

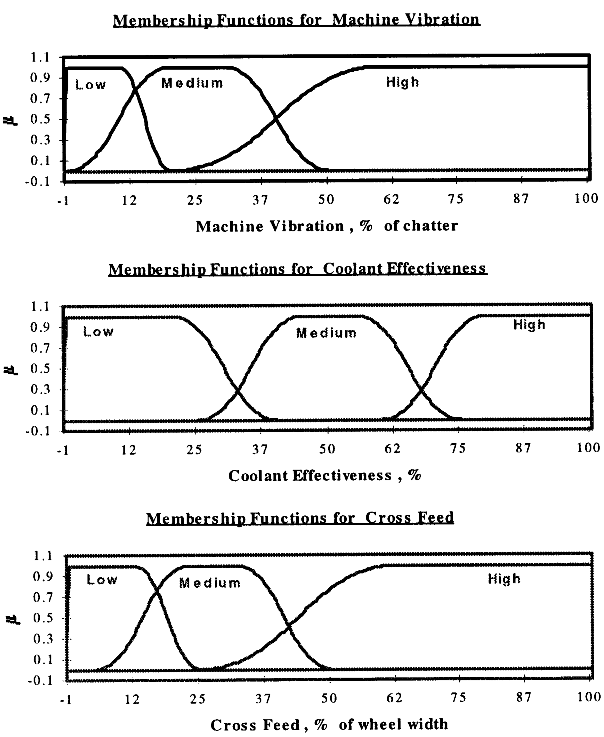

Fig. 4. Definitions of modifier variables

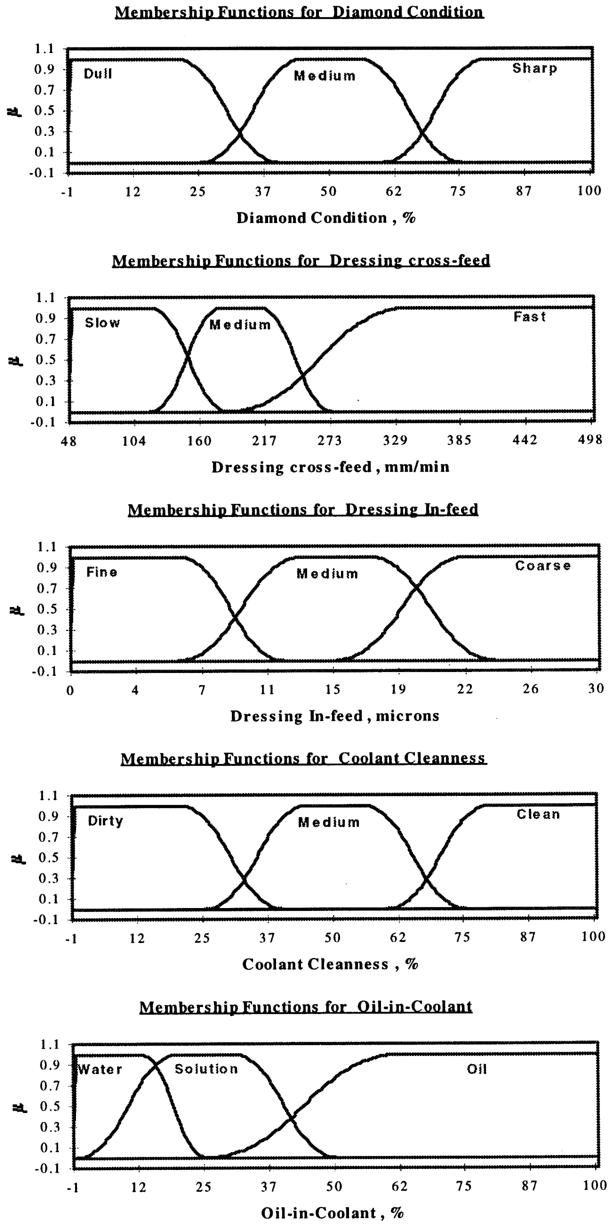

Fig. 5. Definition of auxiliary variables

discrete true/false transition in binary logic. The possibility theory of fuzzy logic provides a measure of the potential ability of a subset in belonging to another subset [15]. It can be shown that probability theory is a special case of possibility theory [16]. Therefore, fuzzy logic has a wider scope and range of applications than many statistical methods.

Engineering applications of fuzzy logic utilize this continuous transition in subset membership to transform a problem from crisp numeric to fuzzy linguistic domains. Instead of operating with numeric values of variables and using mathematical functions to describe relationships, fuzzy logic uses common everyday language to describe variables and uses fuzzy linguistic rules to define relationships. This is particularly advantageous in grinding where some variables, such as grit size, wheel grade, and the effect of coolant, have no precise numeric values. It also enables the use of accumulated knowledge and experience in the form of rules-of-thumb, which cannot be incorporated into a mathematical formula. Yet, the main power of fuzzy logic is that, by proper selection of membership functions and fuzzy rules, it can simulate highly non-linear and complex systems whilst clearly maintaining the physical implications and effects of every variable. A fuzzy variable is defined by the triple [17,18]

$$(U,\ T,\ M) \tag{1}$$

where $U \equiv [UL,\ UU]$, is the universe of discourse or domain of the variable. A variable can be assigned some linguistic values defined by the term set T. M can be semantic rules or mathematical functions that provide the mapping of the variable from U to T and the reverse. If a variable has no crisp numeric values, it can be defined by its linguistic terms only. Then, U can be understood as a mean of sorting linguistic terms relative to each other. A linguistic term, T_i, is defined [17] by the pair:

Table 1
The primary rule-base

#	If	And	And	And Wheel Grade	And Depth of Cut	And Wheel Speed	And Mat. HRc	Then CLA	Wt
1	Table Speed is Very High	Grit Size is Very Coarse	Dressing Coarse	Hard	High	High	Hard	Med.	0.8
2					Med.	High	Hard	Med.	0.5
3						Low	Hard	Coarse	0.8
4				Med.	High	High	Hard	Med.	0.9
5					Med.	High	Hard	Med.	0.5
6						Low	Hard	Med.	0.9
7				Soft	High	Low	Hard	Coarse	0.9
8					Med.	High	Hard	Med.	0.9
9						High	Hard	Coarse	0.9
10						Low	Hard	Coarse	0.5
11			Fine	Hard	High	Low	Soft	Med.	0.8
12					Med.	Low	Soft	Med.	0.5
13				Med.	High	Low	Soft	Med.	0.9
14					Med.	Low	Soft	Med.	0.5
15				Soft	High	High	Soft	Med.	0.9
16						High	Soft	Coarse	0.9
17					Med.	High	Soft	Med.	0.5
18						High	Soft	Coarse	0.5
19						Low	Soft	Med.	0.9
20						Low	Soft	Coarse	0.5
21		Grit Size is Coarse	Coarse	Med.	High	Low	Soft	Med.	0.6
22					Med.	High	Soft	Med.	0.8
23						Low	Soft	Med.	0.5
24				Soft	Med.	Low	Soft	Med.	0.9
25			Fine	Hard	High	High	Hard	Fine	0.3
26				Med.	High	High	Hard	Fine	0.9
27					Med.	High	Hard	Fine	0.5
28						Low	Hard	Fine	0.9
29				Soft	High	Low	Hard	Med.	0.9
30					Med.	High	Hard	Fine	0.9
31						Low	Hard	Med.	0.5
32		Grit Size is Medium	Dressing Coarse	Hard	High	Low	Soft	Med.	0.9
33						Low	Soft	Coarse	0.6
34					Med.	High	Soft	Fine	0.9
35						Low	Soft	Med.	0.5
36						Low	Soft	Coarse	0.5
37				Med.	High	Low	Soft	Med.	0.6
38					Med.	High	Soft	Med.	0.7
39						Low	Soft	Med.	0.5
40				Soft	Med.	Low	Soft	Coarse	0.8
41						Low	Soft	Med.	0.7
42			Dressing Fine	Hard	High	High	Hard	Med.	0.5
43						High	Hard	Fine	0.3
44					Med.	High	Hard	Med.	0.5
45						High	Hard	Fine	0.3
46						Low	Hard	Med.	0.9
47				Med.	High	High	Hard	Fine	0.7
48					Med.	High	Hard	Fine	0.5
49						Low	Hard	Fine	0.7
50				Soft	High	Low	Hard	Med.	0.9
51					Med.	High	Hard	Fine	0.9
52						Low	Hard	Med.	0.5
53	Table Speed is High	Grit Size is Very Coarse	Dressing Coarse	Hard	High	High	Soft	Med.	0.9
54					Med.	High	Soft	Med.	0.5
55						Low	Soft	Med.	0.7
56				Med.	High	High	Soft	Med.	0.9
57					Med.	High	Soft	Med.	0.5
58						Low	Soft	Fine	0.4
59						Low	Soft	Med.	0.4
60				Soft	High	Low	Soft	Med.	0.9
61					Med.	High	Soft	Med.	0.9
62						Low	Soft	Med.	0.5
63			Dressing Fine	Hard	High	Low	Hard	Med.	0.6
64					Med.	High	Hard	Fine	0.9
65						Low	Soft	Med.	0.9
66						Low	Hard	Med.	0.5
67				Med.	High	Low	Hard	Med.	0.8
68					Med.	High	Hard	Fine	0.8
69						Low	Soft	Med.	0.9
70						Low	Hard	Med.	0.5
71				Soft	High	High	Hard	Fine	0.6
72					Med.	High	Hard	Fine	0.5
73						Low	Hard	Fine	0.9
74		Grit Size is Coarse	Coarse	Med.	High	Low	Hard	Fine	0.7
75					Med.	High	Hard	Fine	0.9
76						Low	Hard	Fine	0.5
77				Soft	High	High	Hard	Fine	0.8
78					Med.	High	Hard	Fine	0.5
79						Low	Hard	Fine	0.9
80			Fine	Med.	Med.	Low	Soft	Fine	0.8
81				Soft	High	Low	Soft	Med.	0.9
82					Med.	High	Soft	Fine	0.9
83						Low	Soft	Med.	0.5
84		Grit Size is Medium	Coarse	Hard	High	Low	Hard	Fine	0.7
85					Med.	High	Hard	Fine	0.9
86						Low	Hard	Fine	0.5
87				Med.	High	Low	Hard	Fine	0.7
88					Med.	High	Hard	Fine	0.7
89						Low	Hard	Fine	0.5
90				Soft	High	High	Hard	Fine	0.8
91					Med.	High	Hard	Fine	0.5
92						Low	Hard	Fine	0.7
93						Low	Hard	Med.	0.4
94			Fine	Hard	Med.	Low	Soft	Med.	0.7
95				Med.	Med.	Low	Soft	Fine	0.7
96				Soft	High	Low	Soft	Fine	0.7
97						Low	Soft	Med.	0.7
98					Med.	High	Soft	Fine	0.9
99						Low	Soft	Fine	0.5
100						Low	Soft	Med.	0.5

Table 2
The modifier rule-base

No.	If Modifier variable	Is	Then Surface roughness is estimated by changing primary roughness
1	Machine vibration	Low	No change
2		Medium	One step coarser
3		High	Three steps cover
4	Coolant effectiveness	Low	No change
5		Medium	One step finer
6		High	Two steps finer
7	Cross feed overlay	Low	No change
8		Medium	Half step coarser
9		High	One step coarser
10	Bond type	Vitrified	No change
11		Resinoids	ons step finer
12	Wheel freshness	Continous	One step finer
13		Steady wear	
Compensation for missing primary rules			
14	Table speed	Medium	Equals high
15	Low	Equals high + half step finer	
16	Very low	Equals high + one step finer	
17	Grit size	Fine Medium + one step finer	
18		Very fine	Medium + two steps finer
19	Dressing condition	Medium	Average of fine and coarse
20	Depth of cut	Low	Equals madium + one step coarser
21	Wheel speed	Medium	Average of high and low
22	Material hardness	Medium	Average of Soft and Hard

$$(S,\ P) \tag{2}$$

where $S \equiv [SL, SU]$, $S \subseteq U$, is the supporting subset of T_i, and P is the set of parameters defining the membership function $\mu_{Ti}(y)$ expressing the degree of belonging of the crisp value of y to a specific term T_i, Fig. 1. In the present work, membership functions are defined by five parameters $P = \{a, b, c, d, e\}$, such that:

$$\mu_T(y) = \begin{cases} 0 & y \notin S \\ \frac{1}{2}\left(\frac{y-SL}{a-SL}\right)^e, & y \in [SL, a] \\ 1 - \frac{1}{2}\left(\frac{b-y}{b-a}\right)^e, & y \in [a, b] \\ 1 & y \in [b, c] \\ 1 - \frac{1}{2}\left(\frac{y-c}{d-c}\right)^e, & y \in [c, d] \\ \frac{1}{2}\left(\frac{SU-y}{SU-d}\right)^e & y \in [SU, d] \end{cases} \tag{3}$$

The construction of fuzzy models, selection of membership functions and generation of the rule-base are all based on available experimental knowledge as well as the concept of minimum inference error which has been described in [19]. Once a model is constructed, deduction of information from the model follows three main steps:

(1) Fuzzification of the input data, using Eq. (3). Every crisp value, y, of a variable A is converted into a set of numbers indicating the degree of membership of y into each linguistic term T_i of A.

$$y \equiv \bigcup_{\mathrm{T}}(T_i/\mu_T(y)). \tag{4}$$

For example from Fig. 2, a 1-μm surface roughness can be converted to the set (Medium/0.34, Coarse/0.52): meaning that in a grinding context, this surface roughness is rather high even though it is still within average ranges.

(2) Inference of model predictions using the fuzzy rules. The fuzzy rule base contains a set of rules of the form

$$\text{if } \mathbf{A} \text{ then } \mathbf{B}, \tag{5}$$

where **A** and **B** are fuzzy vectors of propositions concerning input and output variables, respectively. Like most engineering problems, a grinding process model follows a modus ponens rule of inference [14,18], i.e.

$$\mathbf{A}' \bigcap (\bigcup_{\mathrm{i}}(\mathbf{A} \bigcap \mathbf{B})_{\mathrm{i}} \Rightarrow \mathbf{B}' \tag{6}$$

The simplest form of union and intersection operations are the max. and min. operators, respectively [14]. Therefore, inference reduces to the *compositional rule of inference* [17]

$$\mu\ (\mathbf{B}') = \max_i\ (\ \min\ (\ \mu\ (\mathbf{A}'),\ \min_i(\mu(\mathbf{A}),\ \mu(\mathbf{B}))\ )\ ). \qquad (7)$$

Using Eq. (7), one can infer the surface roughness for a given grinding situation.

Table 3
The auxiliary rule-base

No.	If Diamond	And Dressing cross-feed	And Dressing in-feed	Then Dressing condition
1	Sharp	Slow	Fine	Fine
2			Medium	Fine
3			Coarse	Medium
4		Medium	Fine	Fine
5			Medium	Medium
6			Coarse	Medium
7		Fast	Fine	Medium
8			Medium	Coarse
9			Coarse	Coarse
10	Medium	Slow	Fine	Fine
11			Medium	Medium
12			Coarse	Coarse
13		Medium	Fine	Medium
14			Medium	Medium
15			Coarse	Coarse
16		Fast	Fine	Medium
17			Medium	Coarse
18			Coarse	Coarse
19	Dull	Slow	Fine	Medium
20			Medium	Coarse
21			Coarse	Coarse
22		Medium	Fine	Medium
23			Medium	Coarse
24			Coarse	Coarse
25		Fast	Fine	Coarse
26			Medium	Coarse
27			Coarse	Coarse
	Coolant	Oil-mix	Coolant	Coolant effect
28	Off/dry	Any	Any	Low
29	On/dry	Water	Clean	Medium
30			Medium	Low
31			Dirty	Low
32		Solution	Clean	High
33			Medium	Low
34			Dirty	Low
35		Oil	Clean	High
36			Medium	Medium
37			Dirty	Low

(3) Defuzzification of the inferred fuzzy set, $\mathbf{B}'$. This is done by solving Eqs. (3) and (4) backwards from T to U for each fuzzy term and taking the weighted average from all terms [17–19].

2.2. A fuzzy model for surface roughness

Definition of membership functions for CLA surface roughness is shown in Fig. 2 on a natural logarithmic scale. It is based on the fact that the performance of a surface is related to CLA roughness by a geometric progression [11]. Hence, a standard sequence of roughness values would be 0.1, 0.2, 0.4, 0.8, 1.6, 3.2, and 6.4 μm. Therefore, standard surface roughness is fuzzy in nature. For example, a 0.4-μm standard roughness is not a precise number, but means a measured roughness value more than 0.2 μm and less than 0.8 μm. This concept is shown in Fig. 2 where 'Fine' roughness has a nominal value of 0.4 μm but covers the range from 0.2 to 0.6 μm. The membership value, μ, is understood as the degree of the closeness of the measured value to the nominal value of roughness.

The authors have identified 16 variables that can affect surface roughness in grinding, Figs. 3–5. Six of these variables are considered *auxiliary* variables, because their effects can be combined into only two fuzzy variables, namely: 'dressing condition' and 'coolant effectiveness'. These two variables and the remaining ten constitute the set of variables affecting surface roughness. Fuzzy rules describing the effect of each of these 12 variables were obtained from two sources:

(A) Experimental observations for seven *primary* variables based on the work of Farmer et al. [6]. Data showing the variation of surface roughness with (1) table speed, (2) grit size, (3) dressing condition, (4) wheel grade, (5) depth of cut, (6) wheel speed, and (7) work material hardness were used to extract fuzzy rules by the method described in [19].

(B) The remaining five variables are (1) machine vibration, (2) cross-feed, (3) coolant effectiveness, and the two binary variables: (4) bond type (vitrified or resinoid), and (5) wheel freshness (continuous dressing or steady state wear). The effect of these variables on roughness has not been investigated experimentally, mainly because of the difficulty in quantifying them. Therefore, these variables are called *modifier* variables. Fuzzy rules concerning the modifier variables are obtained from available human experience with grinding and from rules-of-thumb and recommendations made by grinding wheel manufacturers, e.g. [8].

Figs. 3 and 4 show membership functions definitions for the seven primary and five modifier variables, respectively, sorted in a descending order of significance. The way the inference engine works is as follows:

(1) Values of the six auxiliary variables are input to the auxiliary rule-base that produces an output estimation of the dressing condition and coolant effectiveness.

(2) Values of the seven primary variables are input to the primary rule-base that produces an output estimation of the surface roughness based on these variables only.

(3) The estimated surface roughness and the five modifier variables are input to the modifier rule-base that produces the final estimation of the expected surface roughness.

In principle, the primary rule-base can contain 6075 rules. However, many of these rules have no practical relevance. The experimental data from [6] have resulted in 495 fuzzy rules, of which only the most significant 100 rules are shown in Table 1. Table 2 shows the rules used by the modifier rule-base. Fig. 5 shows the membership functions definitions for the six auxiliary variables, whilst Table 3 shows the corresponding rules for the auxiliary rule-base.

3. Results and discussion

The operation of the fuzzy model can be best explained by an example. This example is obtained from example *abcdeh* from Table 8.5 in [6]. All of the grinding conditions used in that example are listed below along with the fuzzfication of the crisp values to their linguistic counterparts, according to Figs. 1–5:

1. Table speed: 13 fpm = 4 m min^{-1} = (medium/0.12, high/1.0);
2. Grit size: no. 46 = (medium/0.7, coarse/0.88);
3. Dressing condition: 85% (coarse/1.0), that is because
   - Diamond: (sharp/1.0)
   - Cross-feed: 12 in min^{-1} = 300 mm min^{-1} = (fast/0.85)
   - In-feed: 0.001 in = 25 μm = (coarse/1.0)
   - which result in firing of auxiliary rule no. 9.
4. Wheel grade: J = (medium/0.94, hard/0.32)
5. Wheel speed: 4500 fpm = 23 m s^{-1} = (low/1.0, medium/0.35)
6. Work hardness: 63 HRc = (hard/1.0)
7. Machine vibration: (low/1.0)
   - Coolant effectiveness: (low/1.0), that is because
8. Coolant application: off/dry cut, which fires auxiliary rule no. 28.
9. Cross-feed: 0 = (low/1.0)
10. Bond type: vitrified
11. Wheel sharpness: steady state wheel wear.

Application of the above fuzzy variables to the primary rule base and using Eqs. (6) and (7) results in the firing of primary rules nos. 74, 76, 84, 86, 87, 89; all rules predict 'fine' surface roughness with possibilities 0.88, 0.52, 0.32, 0.32, 0.7, 0.52, respectively. Multiplying possibility of each rule by its weight and collecting the effect of all rules using the union (max.) operation, the model concludes that roughness should be 'Fine' with membership level 0.61 (that is 0.88×0.7 from rule no. 74). Defuzzification of (fine/0.61) from Fig. 2 results in two values, 0.255 or 0.47 μm. However, because of the firing of modifier rules no. 1, 4, 7, 10, 13, and 14, it is concluded that the coarser value should be used. Therefore, the prediction of the fuzzy model is that the surface roughness should be 0.47 μm = 18.5 μin. This is only a 2% error from the actual measured value of 18.1 μin reported in [6].

It is important to note that the performance of the model depends on the accuracy of the rules used in building it. Rules-of-thumb used in the auxiliary and modifier rule bases are context dependent and may not be valid for all grinding situations. For example the rules in Tables 2 and 3 may not be applicable for creep-feed grinding or cases other than surface grinding. Moreover, the primary rules in Table 1 are based on the experimental results reported in [6] which are not ensured to be error-free or not affected by other factors relating to that particular laboratory. Further, the definitions of membership functions in Figs. 3–5 are context dependent and are affected by personal experience that may not be applicable in all grinding situations. Fortunately, the model has been programmed on a spreadsheet using Excel, which makes it very simple to change membership function definitions and various fuzzy rules depending on the particular context. Copies of the program can be obtained by contacting the authors.

4. Conclusions

This paper has presented a complete fuzzy model for prediction of the surface roughness produced by surface grinding operations. Up to 16 independent variables can be used to estimate the surface roughness. All membership functions and the primary, auxiliary, and modifier rule-bases have been presented in detail. The effectiveness and high performance of the model have been demonstrated by a worked example. Moreover, the model is shown to be simpler, more effective, superior in modeling non-linearity, and conceptually clearer than many other approaches.

Acknowledgements

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References

[1] D.J. Whitehouse, Handbook of Surface Metrology, Institute of Physics, Bristol, Bristol, 1994.

[2] S. Malkin, Grinding Technology, Ellis-Horwood, Chichester, 1989.
[3] M. Tawfik, Proc. Int. Conf. AMPT, Dublin City University, Dublin, 1993, p. 1661.
[4] R.P. Lindsay, ASME Paper 72-WA/Prod-13, 1972.
[5] S. Kannappan, S. Malkin, Trans. ASME Eng. Ind. (1972) 833.
[6] D.A. Farmer, J.N. Brecker, M.C. Shaw, Proc. Inst. Mech. Engrs. 182 (3) (1967) 171.
[7] H.K. Tonshoff, J. Peters, I. Inasaki, T. Paul, Ann. CIRP 41 (2) (1992) 677.
[8] Norton Abrasives, Handbook On Grinding, Norton Pty., Sydney, 1992.
[9] L.C. Zhang, T. Suto, H. Noguchi, T. Waida, Int. J. Machine Tools Manuf. 33 (1993) 245.
[10] L.C. Zhang, Int. J. Machine Tools Manuf. 34 (1994) 1045.
[11] M.C. Shaw, Principles of Abrasive Processing, Clarendon Press, Oxford, 1996.
[12] Y.M. Ali, L.C. Zhang, J. Mats. Proc. Tech. 63 (1997) 875.
[13] Y.M. Ali, L.C. Zhang, Proc. 3rd. Int. Conf. Prog. of Cutting and Grinding, Osaka, (1996) 488.
[14] L.A. Zadeh, Fuzzy Logic, Computer, April (1988) 83.
[15] B. Kosko, Fuzzy Thinking, Harper Collins, London, 1994.
[16] B. Kosko, Neural Networks and Fuzzy Systems, Prentice Hall, New Jersey, 1992.
[17] D. Driankov, H. Hellendoorn, M. Reinfrank, An Introduction to Fuzzy Control, Springer-Verlag, Berlin, 1993.
[18] H.J. Zimmermann, Fuzzy Set Theory and its Applications, Kluwer, Boston, 1991.
[19] Y.M. Ali, L.C. Zhang, Fuzzy Sets and Systems (submitted).