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INVESTIGATION OF SHEET METAL FORMING BY BENDING—PART II. PLASTIC WRINKLING OF CIRCULAR SHEETS PRESSED BY CYLINDRICAL PUNCHES

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Abstract—This paper advances a new method of predicting the elastic–plastic bifurcation of plates and shells based on the maDR method and the well-known Lyaponoff's dynamic criterion of stability. This method takes advantage of the character of maDR to overcome the main difficulties in the application of the dynamic criterion naturally. The paper shows that the method is successful in detecting the plastic wrinkling of workpieces in the conical die cup test; hence the paper gives useful results in some detail for manufacturing engineers.

NOTATION

See Ref. [1].

1. INTRODUCTION

The authors have investigated thoroughly the axisymmetric elastic–plastic deformation of workpieces in the conical die cup test—see Fig. 1 in Part I of this study [1]. The results showed that the circumferential membrane force, N_θ, covered a wide band of compression between the area of the ring load circle and the periphery of the workpiece (see Fig. 6 of [1]). When the external load increases continuously, therefore, wrinkling will be the inevitable result. Moreover, the results of [1] also showed that the elements of the workpiece outside the ring load circle had been in the plastic state before wrinkling occurred. Hence, a difficult problem of plastic wrinkling is encountered.

Production engineers have paid great attention to the prediction of wrinkling in sheet metals during forming operations over many decades. A large number of papers have been published. Researchers have discussed the problem from various angles. For example, Geckeler [2], Yu and Johnson [3] and Zhang and Yu [4, 5] have successively investigated the wrinkling of a flange of a circular sheet during its axisymmetric deep-drawing with the aid of a one-dimensional model, an energy method and the approach of combining the Kantorovich and Galerkin methods, respectively. However, researchers have studied wrinkling more frequently by experimental approaches [6, 7]; many problems posed theoretically have been found difficult to solve up to now. For instance, to discuss plastic wrinkling, which is a non-conservative problem, one should use a dynamic criterion. However, most people prefer to use a static or an energy criterion, owing to the difficulties of the former in application. In addition, researchers have found that plastic buckling loads calculated on the basis of the simple J_2 flow theory exceed realistic values, while buckling loads from simple J_2 deformation theory are in good agreement with experimental observations. This represents the so-called plastic buckling paradox found in plates and shells.

The present paper aims to make an investigation of the plastic wrinkling of workpieces in the conical die cup test. It advances a new method of predicting the elastic–plastic bifurcation of plates and shells based on the maDR method and on Lyaponoff's dynamic criterion for stability. This method takes advantage of the character of maDR to overcome the main difficulties found in the application of the dynamic criterion in a natural manner. Examples show that the method is successful in detecting the plastic wrinkling of workpieces in the conical die cup test, and hence it gives useful results in some detail for production engineers.

2. DETECTION OF WRINKLING LOADS

Many difficulties usually exist in the applicaton of the dynamic criterion. The main problems are that

(i) an initial value problem has to be treated when one uses the dynamic criterion, while it is difficult for the usual methods of solving static problems to be used in this function. Furthermore, the criterion relates to any initial disturbances as well as to the bounded property of the mechanical system discussed when $t \to \infty$;

(ii) various movements due to disturbances will produce complexity of loading and unloading.

Fortunately, the maDR method can be naturally combined with the dynamic criterion. Firstly, the maDR method itself solves static problems by transforming them into corresponding dynamic ones. Hence, there does not exist any difficulty in producing dynamic disturbances. Moreover, the dynamic disturbances possessed by the maDR method are free, and one can observe the bounded property of a system over a long period of time. Secondly, one should first overcome the difficulties of loading and unloading during iteration as maDR is chosen to solve an elastic–plastic problem, so that the above problem (ii) is also resolved. Details and characteristics of the maDR method can be found in [1, 8].

From Fig. 1, one can understand exactly how the maDR method works in detecting a bifurcation point. In Fig. 1, curve I stands for the elementary equilibrium path of a mechanical system and curve II is the secondary path after bifurcation. Every point on these two curves stands for a stable equilibrium state of the system, but that on the dotted curve is an unstable one. Therefore, when the state of the system is represented by a point on curve I, the fictitious vibration of the maDR method is also stable (such as point B_0 on curve I). With the increase of deformation, the state point of the system goes along curve I. However, if it goes beyond the bifurcation point A and, for example, reaches point B, the fictitious vibration of maDR becomes unstable, and a jump from B to C occurs; another stable vibration around C then starts. Once two points like C are detected, the bifurcation point A can be obtained approximately by linear extrapolation.

It is easy to write an algorithm for detecting the plastic wrinkling for workpieces in the conical die cup test according to the above statement. Obviously, if the axisymmetric equilibrium state before wrinkling is stable, the circumferential incremental displacement in the present increment cycle of external load should satisfy the condition $\delta v = 0$. Therefore, if $\delta v \geqslant \bar{v}$ ($\bar{v}$ is a small given number; the present paper takes $\bar{v} = 10^{-3}$) in a certain load increment (δq) cycle, wrinkling occurs. This point is then recorded and $\delta q^{(1)} = \lambda^{(1)} \delta q$

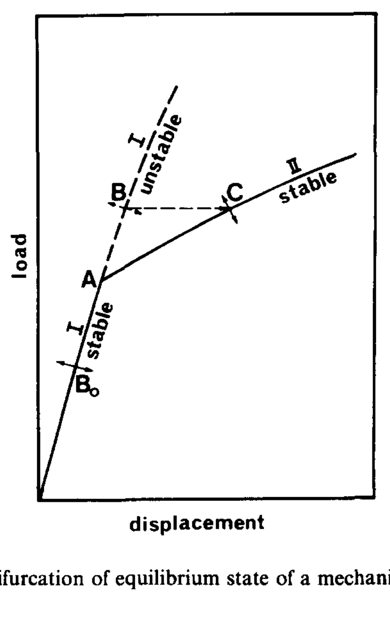

FIG. 1. Bifurcation of equilibrium state of a mechanical system.

($\lambda^{(1)} < 1$) is taken for a new calculation. If $\delta v^{(1)} \geqslant \bar{v}$, the approximate wrinkling point is obtained by extrapolation from these two points. Otherwise, we take $\delta q^{(2)} = \lambda^{(2)}\delta q$ ($\lambda^{(2)} > \lambda^{(1)}$) and calculate again until $\delta v^{(j)} \geqslant \bar{v}$. Figure 2 shows the detailed flow chart of the above algorithm.

3. ANALYSIS OF EXAMPLES

The model and material behavior of the workpieces, and other conditions, such as friction, are the same as those in [1], but the elementary equations are different. In order to observe wrinkling that is non-axisymmetric, one must use the following equations.

3.1. *Equilibirum equations*

$$\begin{cases} \dfrac{\partial \delta N_r}{\partial r} + \dfrac{1}{r}\dfrac{\partial \delta N_{r\theta}}{\partial \theta} + \dfrac{1}{r}(\delta N_r - \delta N_\theta) = 0, \\ \dfrac{\partial \delta N_{r\theta}}{\partial r} + \dfrac{1}{r}\dfrac{\partial \delta N_\theta}{\partial \theta} + \dfrac{2}{r}\delta N_{r\theta} = 0, \\ \dfrac{\partial^2 \delta M_r}{\partial r^2} + \dfrac{2}{r}\dfrac{\partial \delta M_r}{\partial r} - \dfrac{1}{r}\dfrac{\partial \delta M_\theta}{\partial r} + \dfrac{2}{r}\dfrac{\partial^2 \delta M_{r\theta}}{\partial r \partial \theta} + \dfrac{2}{r^2}\dfrac{\partial \delta M_{r\theta}}{\partial \theta} \\ \quad + \dfrac{1}{r^2}\dfrac{\partial^2 \delta M_\theta}{\partial \theta^2} + \left\{ \dfrac{\partial^2 \delta w}{\partial r^2}(N_r + \delta N_r) + \dfrac{\partial^2 w}{\partial r^2}\delta N_r + \dfrac{2}{r}\left[\dfrac{\partial^2 \delta w}{\partial r \partial \theta}(N_{r\theta} \right.\right. \\ \quad \left. + \delta N_{r\theta}) + \dfrac{\partial^2 w}{\partial r \partial \theta}\delta N_{r\theta}\right] - \dfrac{2}{r^2}\left[\dfrac{\partial \delta w}{\partial \theta}(N_{r\theta} + \delta N_{r\theta}) + \dfrac{\partial w}{\partial \theta}\delta N_{r\theta}\right] \\ \quad \left. + \left(\dfrac{1}{r}\dfrac{\partial w}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2 w}{\partial \theta^2}\right)\delta N_\theta + \left(\dfrac{1}{r}\dfrac{\partial \delta w}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2 \delta w}{\partial \theta^2}\right)(N_\theta + \delta N_\theta) \right\} + \delta q = 0. \end{cases}$$

3.2. *Geometrical relations*

$$\begin{cases} \delta \varepsilon_r^0 = \dfrac{\partial \delta u}{\partial r} + \dfrac{\partial w}{\partial r}\dfrac{\partial \delta w}{\partial r} + \dfrac{1}{2}\left(\dfrac{\partial \delta w}{\partial r}\right)^2, \\ \delta \varepsilon_\theta^0 = \dfrac{\delta u}{r} + \dfrac{1}{r}\dfrac{\partial \delta v}{\partial \theta} + \dfrac{1}{r^2}\dfrac{\partial w}{\partial \theta}\dfrac{\partial \delta w}{\partial \theta} + \dfrac{1}{2}\left(\dfrac{1}{r}\dfrac{\partial \delta w}{\partial \theta}\right)^2, \\ \delta \gamma_{r\theta}^0 = \dfrac{1}{r}\dfrac{\partial \delta u}{\partial \theta} + \dfrac{\partial \delta v}{\partial r} - \dfrac{\delta v}{r} + \dfrac{1}{r}\dfrac{\partial \delta w}{\partial r}\left(\dfrac{\partial w}{\partial \theta} + \dfrac{\partial \delta w}{\partial \theta}\right) + \dfrac{\partial w}{\partial r}\left(\dfrac{1}{r}\dfrac{\partial \delta w}{\partial \theta}\right), \\ \delta K_r = -\dfrac{\partial^2 \delta w}{\partial r^2}, \\ \delta K_\theta = -\left(\dfrac{1}{r}\dfrac{\partial \delta w}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2 \delta w}{\partial \theta^2}\right), \\ dK_{r\theta} = -\dfrac{1}{r}\dfrac{\partial^2 \delta w}{\partial r \partial \theta} + \dfrac{1}{r}\dfrac{\partial \delta w}{\partial \theta} \end{cases}$$

and

$$\begin{cases} \delta \varepsilon_r = \delta \varepsilon_r^0 + z\,\delta K_r, \\ \delta \varepsilon_\theta = \delta \varepsilon_\theta^0 + z\,\delta K_\theta, \\ \delta \gamma_{r\theta} = \delta \gamma_{r\theta}^0 + 2z\,\delta K_{r\theta}. \end{cases}$$

3.3. *Constitutive equations*

As before [1], the simple J_2 flow theory is used, i.e.

in the elastic region:

$$\delta \varepsilon_{ij} = \frac{\delta \sigma_{ij}}{2G} - \frac{v}{E}\delta \sigma_{kk}\delta_{ij};$$

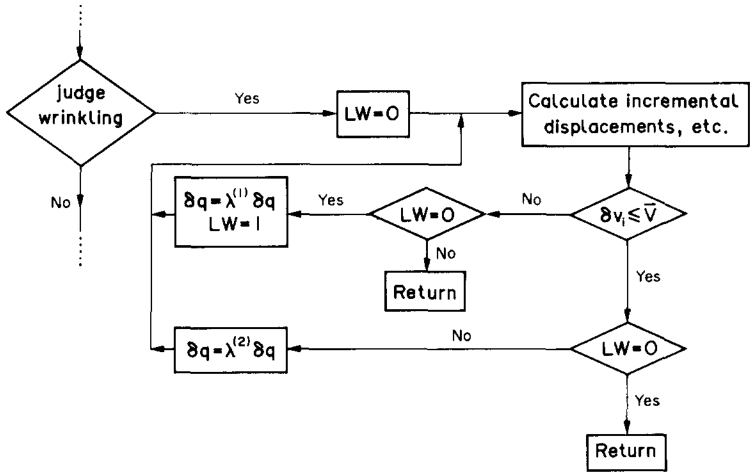

FIG. 2. Flow chart of the algorithm for detecting the plastic wrinkling of the workpieces in the conical cup test.

in the plastic region:

$$\delta e_{ij} = \frac{\delta s_{ij}}{2G} + \frac{\partial \phi}{\partial \sigma_{ij}} \delta\lambda,$$

$$\delta\varepsilon_{kk} = \frac{1-2v}{E}\delta\sigma_{kk},$$

$$\text{and } \delta\lambda = \begin{cases} 0, & \text{when } \phi(\sigma_{ij} + \delta\sigma_{ij}) \leqslant 0 \\ h^* \delta\phi & \text{when } \phi(\sigma_{ij} + \delta\sigma_{ij}) > 0, \end{cases}$$

where v is Poisson's ratio, E is Young's modulus, G is the shear modulus, e_{ij} are the deviatoric strain components, s_{ij} are the deviatoric stress components, ϕ is the Mises loading function and h^* is the work-hardening modulus.

When the above elementary equations are changed into corresponding finite difference equations, the wrinkling loads for workpieces can be calculated very conveniently by the algorithm described by Fig. 2.

Comparisons of the present theoretical results with experimental ones are shown in Fig. 3(a, b). For the workpiece 150-2.0-C45 (this stands for a workpiece of diameter 150 mm and thickness 2 mm pressed by a cylindrical punch with diameter 45 mm; detailed explanations are in [1]), the experimental wrinkling load is 900 kg, while the theoretically predicted load is 925 kg. For 150-2.0-C65, the former is 1000 kg and the latter 1031.25 kg. Errors in these two cases are only 2.78% and 3.13%, respectively. It is significant that the present theoretical wrinkling loads are obtained by using the simple J_2 flow theory, so that the present results are contrary to the plastic buckling paradox. There is no doubt that the method and the results in the present paper provide a new approach and new theoretical materials for the further study of the paradox.

Figure 4(a) shows that the circumferential wrinkling mode of the workpiece oscillates like a sine curve. Theoretical calculations yield four waves, which accords exactly with experimental observations. It follows that the wrinkling mode of four waves corresponds to the critical wrinkling load for a workpiece in the conical die cup test. It will be very useful for researchers to carry out approximate analytical solutions of similar problems.

It should be pointed out that the present method can be used directly to investigate post-wrinkling deformation after the detection of the wrinkling load. Hence, one now can analyse a problem thoroughly from the elementary deformation state (i.e. the state before bifurcation) and the wrinkling (or buckling, or bifurcation) prediction to post-wrinkling deformation by a unified method. This advantage makes the corresponding computer program simple and efficient.

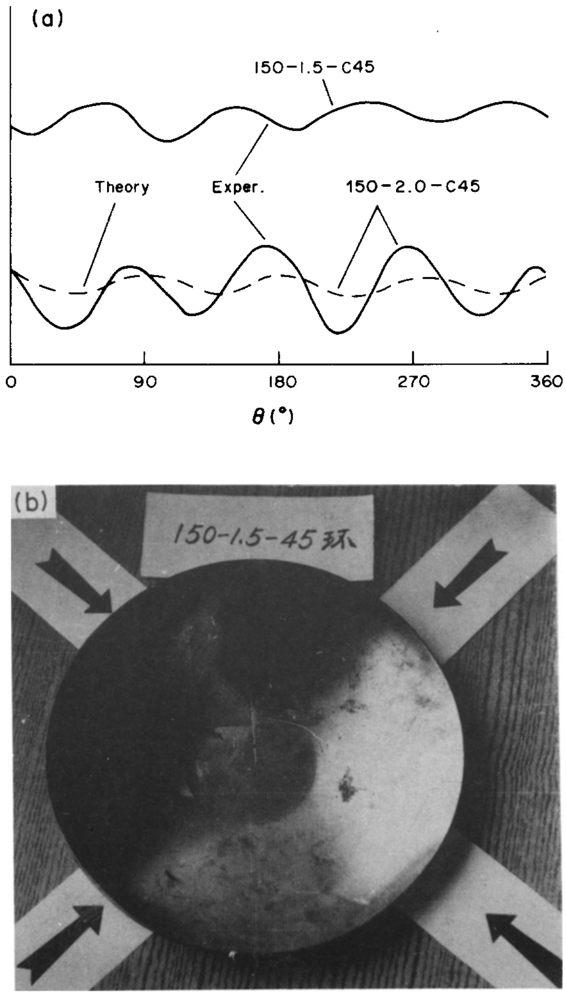

FIG. 4. The wrinkling mode of the workpieces: (a) circumferential shapes of the wrinkling; (b) photograph of 150-1.5-C45 after wrinkling.

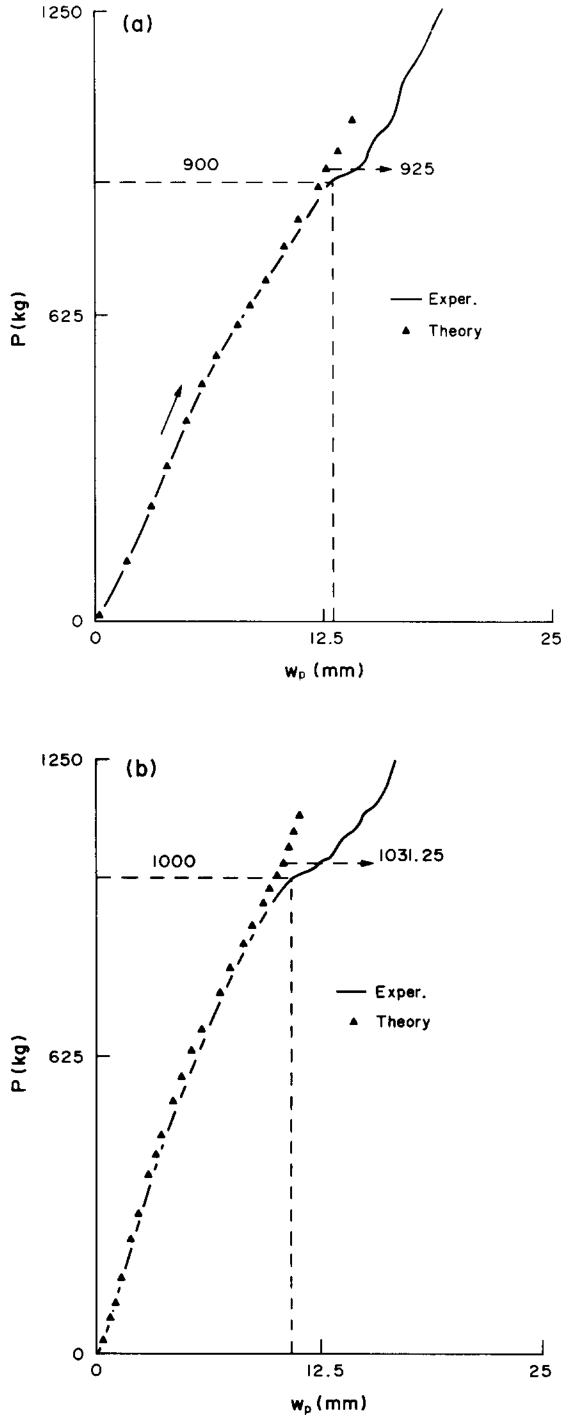

FIG. 3. The external load (P)–punch displacement (w_p) curves: (a) 150-2.0-C45; (b) 150-2.0-C65.

Figure 4(b) is a photograph of workpiece 150-1.5-C45 after wrinkling. The four-wave wrinkling mode is clearly shown. It shows that the peaks of the wrinkling waves in the radial direction are not at the periphery, but inside it. The authors think that this phenomenon is very much related to the fact that the position of the maximum circumferential membrane force before wrinkling does not stay at the periphery but moves to the interior as the external load increases to a certain level (cf. Fig. 6 of [1]). Hence, one should consider the shape of the wrinkling mode in the radial direction in order to obtain a more reasonable solution for the problem from an approximate analytical approach.

4. CONCLUSIONS

(1) The new method of detecting elastic–plastic wrinkling (or bifurcation) of sheet metals by combining the maDR method with a dynamic criterion possesses the advantages of convenient applications and high accuracy, and can be widely used to analyse the bifurcation of more complicated plate-shell structures. It also provides new theoretical results for further study of the plastic buckling paradox.

(2) The maDR method makes it possible to analyse a problem thoroughly from its elementary deformation state and wrinkling to post-wrinkling deformation by a unified approach.

(3) The wrinkling mode of the four waves corresponds to the critical load of the workpieces in the conical die cup test.

(4) The circumferential shape of the wrinkling mode of such a workpiece can be approximated by a sine curve in engineering analyses, but an appropriate radial shape of the mode should also be considered in order to obtain more reasonable results.

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