Int. J. Mach. Tools Manufact. Vol. 33, No. 4, pp. 587–597, 1993.
Printed in Great Britain

0890–6955/93$6.00 + .00

APPLIED MECHANICS IN GRINDING—III. A NEW FORMULA FOR CONTACT LENGTH PREDICTION AND A COMPARISON OF AVAILABLE MODELS

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(*Received* 3 *January* 1992; *in final form* 1 *July* 1992)

Abstract—The contact length between a grinding wheel and a workpiece during operation is one of the principal factors that specify thermal and mechanical deformation of the workpiece. In this paper, we first propose a new formula for the contact length prediction, present a simple criterion to determine the applicable range of any approximate formulae, and then carefully compare our formula with available models, discuss their characteristics and limitations in application. Our comparison shows that the present formula is excellent in fitting experimental results from different types of operations that cover conventional and creep feed grinding. Moreover, it only needs a small number of input parameters, i.e. the normal grinding force, the depth of cut, the elasticity of grinding wheel and a condition coefficient, which are easily obtained in a grinding test.

NOMENCLATURE

D_g	mean diameter of grains of a wheel
d_c	real depth of wheel cut
d_n	nominal depth of wheel cut
E_e	equivalent elastic modulus (see Ref. [28])
E_g	elastic modulus of grains
E_s	elastic modulus of a grinding wheel
E_w	elastic modulus of the workpiece material
F	reference tangential force as $t_e=1$ (μm) (see Refs [9, 37])
F'_n	normal grinding load per unit wheel width
H	hardness grade of a grinding wheel, each grade corresponds to a digital number, e.g. $H = 0$, $I = 1$, $J = 2$, $K = 3$, $L = 4$, $M = 5$, etc. (see Refs [30, 38])
K	elastic bulk modulus of wheel–workpiece combination (see Ref. [6])
K_g	defined as $(1-\nu_g^2)/E_g$
K_s	defined as $(1-\nu_s^2)/E_s$
K_w	defined as $(1-\nu_w^2)/E_w$
L_c	modified contact length
L_g	geometrical contact length
M	number of active grains per square millimetre of a wheel surface (see Ref. [4])
N	a finite constant (see equation (9))
Q_w	material removal rate
q	speed ratio of V_s to V_w
R_0	equivalent wheel radius before deformation
R_d	equivalent wheel radius after deformation
r	surface roughness from peak to valley (see Refs [13, 21])
S	the structure number of a grinding wheel = 4, 6, 8, etc (see Refs. [30, 38])
t	average chip thickness (see Ref. [6])
t_e	equivalent grinding thickness, defined as $(\mu F'_n/F)^{1/\zeta}$ (see Refs [9, 38])
V	volume percent of bond material in a wheel (see Ref. [31])
V_s	peripheral speed of a wheel
V_w	table speed
$w(x)$	normal displacement of a wheel at x (see equation (1) and Fig. 1)
x, y	coordinates (defined by Fig. 1)
α_i	defined by equation (8)
δ	the mutual approach of remote points in grain and workpiece (see Ref. [6])
ζ	condition coefficient (defined by formula (6))
$\zeta_1, \zeta_2, \zeta_3$	constants determined by measured results
ϑ	increment of working engagement due to local thermal deflections (see Refs [32, 37])

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μ	ratio of normal grinding force to tangential grinding force (see Refs [1, 9, 37])
ν_g	Poisson's ratio of grains
ν_s	Poisson's ratio of a wheel
ν_w	Poisson's ratio of a workpiece
ξ	defined by equation (3)

1. INTRODUCTION

THE CONTACT length between a grinding wheel and a workpiece during operation plays an extremely important role in studying the surface integrity of the workpiece. It is one of the principal factors that contribute to thermal and mechanical deformation because it determines the bottom length of heat source and interface force distributions (Zhang *et al.*[1]). Owing to the complexity of the interface condition during grinding, it is rather difficult to investigate this problem strictly by analytical methods. Great effort has been made to understand the mechanism of the contact deformation between a wheel and a workpiece and to produce practical models for the contact length calculation. Investigations into this problem could be classified in different ways. From the viewpoint of their research scales, they may be divided into microscopic and macroscopic methods. By the means of research employed, however, they could be categorized into experimental, experimental/analytical and numerical approaches.

Most of the previous studies have been of an experimental nature. Peklenik [2] was the first to apply a thermocouple method to measure the contact length macroscopically. By using Peklenik's method, Makino *et al.* [3] observed that the real contact length could be up to twice the geometrical. Some others announced, however, that the measured lengths could range between one and ten times the geometrically calculated lengths (e.g. Brown *et al.* [4], Hahn and Lindsay [5], Sauer and Shaw [6] and Snoeys and Wang [7]). It was realized that grinding conditions have a significant effect on the size of this interface zone. Depth of cut, coolant chemistry and its application method, wheel and work speeds as well as their mechanical and thermal properties were found to be the three important groups of parameters that influence the actual contact arc. Based on Verkerk's experimental results [8, 9], Maris [10] (see also Snoeys *et al.* [11]) proposed an empirical formula to calculate the real contact length, where the depth of cut and the speed ratio of wheel/work were involved. The applicable range of such a formula was narrow, however. This reminded people that the relation between grinding conditions and the variation of contact length is complex and that one needs first to understand the principle of the deformation of a wheel–workpiece system.

Microscopic research has been emphasized since the 1950s (for example, Backer and Merchant [12], Brandin [13], Gu and Wager [14, 15], Hahn [16], Harris and Lavine [17], Ramanath and Shaw [18], Saini [19, 20], Salje *et al.* [21, 22], Sedriks and Mulhearn [23], Torrance [24], Wager and Gu [25] and Wager and Saini [26]). This approach tries first to observe the details of the interaction between individual grains and a workpiece surface, such as the effects from local deflection of grains, grain shape, chip formation, local heat transfer, surface roughness of the workpiece and so forth, and then to combine these effects comprehensively to produce realistic formulae for contact length prediction. However, it is not easy to do so. That is why only two formulae, to the authors' knowledge, have been proposed by this approach and only the effect of surface roughness of a workpiece has been involved (Brandin [13] and Salje *et al.* [21]). Numerical analysis could take many principal factors into account simultaneously (König and Steffens [27] and Zhang *et al.* [1]) so that it may yield a more accurate result. Unfortunately, it is not so convenient or efficient from the standpoint of research engineers because it requires too many input data that should be changed when any working condition is changed, and needs too long a time to complete an analysis even when a powerful computer is employed.

A most common and most acceptable approach seems to be the experimental/analytical method, which considers the grinding wheel as a continuum disc and deduces approximate formulae under various hypotheses (e.g. Aerens [28], Brown *et al.* [4], Kumar and Shaw [29], Lindsay [30, 31], Quiroga [32], Sauer and Shaw [6], Snoeys and Wang [7] and Verkerk [9]). However, the assumptions applied, which were usually necessary to lead to an explicit analytical expression, also brought about a limit on the range of validity of the formula. We can say that at present there does not exist a general formula that can cover most grinding cases, although all researchers showed that their formulae were supported by their experimental data.

It is the purpose of this paper to produce a new formula for the contact length prediction, which has a simple form but has the ability to cover a relatively wide range of grinding conditions. This formula is actually based on our previous macro-deformation model [1]. Particular treatment is also given to making a careful comparison of available models so that grinding engineers can easily understand the advantages, disadvantages and application limits of those formulae. A criterion for determining the applicability of any formulae is then presented to meet such a requirement. Our comparison shows that the present formula is excellent in fitting experimental results from different types of operations that covered conventional and creep-feed grinding. Moreover, it only needs a few input parameters, i.e. the normal grinding force, the depth of cut, the elasticity of the grinding wheel and a condition coefficient, which are easily obtained from a practical operation.

2. FORMULATION

Zhang *et al.* [1] have shown that macro-deformation is the most important factor among numerous parameters that contribute to the variation of interface contact zone. The reason is straightforward. The total deformation of a wheel–workpiece system is mainly the result of mechanical and thermal distortion. Thermal effects on the wheel could partly be reflected by using the macroscopically measured elastic modulus of the wheel (Zhang *et al.* [1, 33], Tanaka *et al.* [34] and Nakayama [35]), as the modulus, in turn, will change the amount of wheel deflection. Interface force between the wheel and the workpiece, on the other hand, is another variable that, to a great extent, expresses the resultant influence from wheel sharpness, wear rate, properties of workpiece material, coolant chemistry and so on. It is then reasonable to expect that an approximate formula obtained from an overall macro-deformation analysis may predict realistically the variation of contact length, if it includes explicitly Young's modulus, interface force, depth of wheel cut and another carefully arranged empirical parameter. This empirical parameter, hopefully, can respond to those effects that have not been reflected by the first three variables, and can partly compensate the loss in accuracy owing to various assumptions.

Keeping these in mind, let us consider a wheel–workpiece system with initial wheel radius R_0, as shown in Fig. 1. Assume that the deformed wheel is still a circular cylinder, but its radius has been changed into R_d, and that the size of contact length

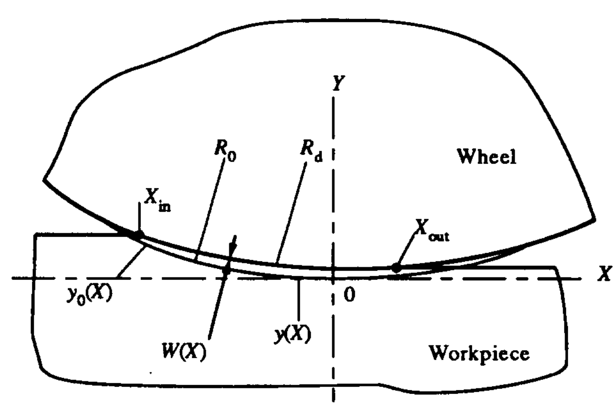

FIG. 1. Interface deformation of a wheel–workpiece system.

is small compared with the wheel size.† The deformed wheel profile in the contact gap, therefore, could approximately be expressed as:

$$y(x) = y_0(x) + w(x). \tag{1}$$

Differentiating equation (1) twice, we get:

$$\frac{d^2 y(x)}{dx^2} - \frac{d^2 y_0(x)}{dx^2} = \frac{d^2 w(x)}{dx^2} \tag{2}$$

where the left-hand side of the equation represents the curvature difference of the wheel profiles before and after deformation. Alternatively, we know that the contribution of shear interface force to $w(x)$ is negligible (Zhang *et al.* [1]), and that:

$$\frac{d^2 w(x)}{dx^2} = -\frac{16(1-\nu_s^2)F_n'}{\pi E_s(x_{out}-x_{in})^2} \equiv -\xi \tag{3}$$

from the analysis of elasticity when we assumed that the pressure over interval $[x_{in}, x_{out}]$ was elliptical (see, e.g. Ref. [36]), where F_n' is the normal grinding force per unit wheel width, ν_s is Poisson's ratio of the grinding wheel, and E_s is the macroscopic elastic modulus of the wheel that could include thermal effect (Zhang *et al.* [1] and Tanaka *et al.* [34]) and microscopic effect of complex wheel components (Zhang *et al.* [1, 33]). A substitution of equation (3) into equation (2) leads to:

$$\frac{1}{R_d} - \frac{1}{R_0} = -\xi \tag{4}$$

and this, in turn, brings about:

$$R_d \approx R_0\left(1 + \frac{8(1-\nu_s^2)F_n'}{\pi E_s d_c}\right). \tag{5}$$

It should be recalled that expression (5) was obtained under certain hypotheses of macro-deformation of the grinding wheel. Many other factors, like surface roughness, coolant feature and its supply manner, dependence of wheel and workpiece elasticity upon temperature and so on, which also influence wheel deformation, have not been involved directly. It is then necessary to introduce another parameter to reflect the comprehensive effect of these factors so that the formula can be used in a wide range of grinding conditions. A simple way is to change equation (5) into:

$$R_d = R_0\left(1+\zeta\frac{(1-\nu_s^2)F_n'}{E_s d_c}\right) \tag{6}$$

where ζ is a constant determined by one set of measured data for a class of grinding operations. Equation (6) can easily be used to predict the contact length, L_c, for the same class of operations, i.e.

$$L_c = R_d \arccos\left(1-\frac{d_c}{R_d}\right). \tag{7}$$

† Figure 1 is the case of surface grinding. For other types of grinding operations, the following analysis is still valid provided that R_0 and R_d are replaced by corresponding equivalent radii.

3. A COMPARISON OF AVAILABLE MODELS

3.1. *Criterion of validity and conditions of evaluation*

At least 14 approximate formulae, to the authors' knowledge, have been proposed for contact length calculation in the last few decades. If we rewrite them into a unified non-dimensional form of

$$\frac{L_c^{(i)}}{L_g} = \alpha_i \qquad (i = 1, \dots, 14) \tag{8}$$

where L_g is the geometrically calculated contact length, condition

$$1 \leq \alpha_i < N \tag{9}$$

should always be satisfied (where N is a finite constant). Any formula which violates equation (9) in a certain range of grinding conditions must be wrong in that range. Inequality in equation (9) is therefore the criterion of application limit of any possible formulae (without considering their accuracy of prediction). Range of application, fitting ability to measured results, convenience of input data preparation, dependence upon pre-measured results and ease of computation are the terms to evaluate the advantages, or disadvantages, of the formulae in this paper. A formulae with wider applicable range, higher fitting ability to various experimental data, easier input, simpler form and less dependence on extra-experimental data is a better formula.

3.2. *Applicable range*

Table 1 gives a list of available formulae and shows briefly the main ideas of their proposers in derivation. It is obvious that formulae $(\dot{1})$, and $(\dot{4})$ and $(\dot{6})$–$(\dot{1}2)$ satisfy the left-hand side of equation (9) in any grinding conditions since they have the form of $\alpha_i = 1+\Delta_i$ with $\Delta_i \geq 0$. However, formulae $(\dot{2})$, $(\dot{3})$, $(\dot{5})$, $(\dot{1}3)$ and $(\dot{1}4)$ do not have such an explicit form so that particular attention is required to check their validities before application. Under the grinding conditions shown in Table 2 (from Vansevenant [37]), for example, the applicable ranges of formulae $(\dot{2})$, $(\dot{3})$ and $(\dot{1}4)$ are very narrow (see Fig. 2) since $\alpha_i \leq 1$ occurs in most of the cases. Here, formula $(\dot{1}4)$ is not valid as $d_c \geq 0.019$ (mm), formula $(\dot{2})$ becomes ridiculous when $d_c \geq 0.006$ (mm), while formula $(\dot{3})$ is only valid for $d_c \leq 0.0026$ (mm). In addition, the prediction accuracy of these formulae are very poor even in their valid ranges.

It is not easy to verify the validity of the right-hand side of inequality (9) for all kinds of formulae. However, those formulae with ratio $(F_n')/(d_c)$, like $(\dot{1})$, $(\dot{8})$, $(\dot{1}2)$ and $(\dot{1}3)$, will satisfy the condition because the speed that F_n' approaches zero is usually much faster than that of d_c (see Fig. 3). The verification of formulae $(\dot{2})$, $(\dot{3})$ and $(\dot{7})$ can be carried out in a similar way. Take formula $(\dot{7})$ as an example. The formula will be fine if the speed that $(F_n')^{\xi_2}$ approaches zero is faster than that of $(d_c)^{\xi_3}$ mathematically. Formula $(\dot{1}4)$ also satisfies the condition because the term $\ln(d_s)$ appears in a negative exponent. The condition of the right-hand side of equation (9), however, is less important than its left-hand side.

3.3. *Convenience and accuracy*

Formula $(\dot{5})$ is non-linear with respect to L_c so that it is not convenient in application. An iteration procedure is needed. Furthermore, it is not easy to estimate properly the number of active grains, M, in the expression. Formulae $(\dot{9})$ and $(\dot{1}1)$ are very similar in their forms although they were derived from very different standpoints. The value of ϑ in $(\dot{1}1)$, unfortunately, is not easy to measure in practice. This limits its application. Formula $(\dot{1}0)$, however, is only part of $(\dot{9})$. A question immediately arises from such a comparison: what is the function of the last term $(r/d_c)^{1/2}$, in formula $(\dot{9})$? The examination by using the data listed in Table 2 (see Fig. 5), shows that such a term

TABLE 1. A LIST OF APPROXIMATE FORMULAE FOR MODIFIED CONTACT LENGTH CALCULATION

i	α_i	Source	Basis
$(\dot{1})$	$\left\{1+\zeta\frac{(1-\nu_s^2)F_n'}{E_s d_c}\right\}\frac{\arccos\left(1-\frac{d_c}{R_d}\right)}{\arccos\left(1-\frac{d_n}{R_0}\right)}$	From equation (7) of the present paper	Elastic deformation, improved by one empirical parameter
$(\dot{2})$	$10^{-3}\left\{\frac{F_n'}{(2R_0)^{1/2}d_c^{3/2}D_g^2(1.33H+2.2S-8)}\right\}^{1/3}$	Lindsay [30], see also Lindsay and Hahn [38] and Des Ruisseaux and Zerkle [39]	Spring system deformation model
$(\dot{3})$	$\frac{0.33(F_n')^{1/3}}{(2R_0)^{1/6}d_c^{1/2}(44.6-V)}$	Lindsay [31], see also Hahn [40, 41]	Elastic deformation of the wheel
$(\dot{4})$	$\left\{1+\zeta_1\frac{t_c^{\zeta_2}}{q}\right\}^{1/2}$	Verkerk [9]	Wheel deformation with modification from two empirical parameters
$(\dot{5})$	$2\left\{\frac{\left[\frac{9}{8D_g}(K_w+K_g)^2\right]^{1/3}}{d_c}\right\}^{1/2}\left\{\frac{F_n'}{ML_c}\right\}^{1/3} + 1.6\left\{\frac{(K_w+K_s)F_n'}{d_c}\right\}^{1/2}$	Brown *et al.* [4]	Separately consider grain–work and wheel–work deformation
$(\dot{6})$	$\left\{1+4\frac{KF_n'+(\delta-t)}{d_c}\right\}^{1/2}$	Sauer and Shaw [6]	Wheel–workpiece deformation
$(\dot{7})$	$\left\{1+\zeta_1\frac{(F_n')^{\zeta_2}}{(d_c)^{\zeta_3}}\right\}^{1/2}$	Sauer and Shaw [6]	Wheel deformation, modified by three undetermined parameters
$(\dot{8})$	$\left(1+\zeta\frac{F_n'}{E_e d_c}\right)^{1/2}$	Aerens [28]	Elastic deformation, improved by one empirical parameter
$(\dot{9})$	$\left(1+\frac{r}{d_c}\right)^{1/2}+\left(\frac{r}{d_c}\right)^{1/2}$	Brandin [13]	Roughness of the workpiece surface
$(\dot{10})$	$\left(1+\frac{r}{d_c}\right)^{1/2}$	Salje *et al.* [21]	Roughness of the workpiece surface
$(\dot{11})$	$\left(1+\frac{\vartheta}{d_c}\right)^{1/2}+\left(\frac{\vartheta}{d_c}\right)^{1/2}$	Quiroga [32]	Local thermal expansion at inlet and outlet of the contact zone
$(\dot{12})$	$\left\{1+\frac{0.19(1+\nu_s)R_0F_n'}{E_sL_gd_c}\right\}^{1/2}\cdot\left\{1+\frac{2F_n'\ln(L_g/2)}{\pi E_w d_c}\right\}^{-1/2}$	Kumar and Shaw [29]	Separately consider grain–work and wheel–work deformation
$(\dot{13})$	$4\left\{(K_w+K_s)\frac{F_n'}{\pi d_c}\right\}^{1/2}$	Snoeys and Wang [7]	Matress model for wheel deformation
$(\dot{14})$	$4.95q^{-0.216}\exp\{-0.0205(q^{0.33})\ln(d_c)\}$	Maris [10]	Regression of experimental data

makes a big difference to the results. It then follows that formula $(\dot{10})$ is not a good approximation.

A careful comparison has been made for our new formula, equation (7) or $(\dot{1})$ in Table 1, with numerous experimental data that covered conventional grinding and

TABLE 2. EXPERIMENTAL MEASUREMENTS [37]

d_c (μm)	V_w (mm/s)	F'_n (N/mm)	r (μm)	α
5	400	3.3	7.5	2.9
10	200	3.5	9.5	2.6
20	100	4.3	8.8	2.05
40	50	5	5	1.65
80	25	6	5.5	1.4
160	12.5	6	4	1.25
320	6.25	7.6	3	1.15
640	3.125	10	2	1.1

V_s = 30 m/s; Q_w = 2 mm³/mm s; E_s = 30 kN/mm²; E_w = 210 kN/mm².
Wheel: EK46F12ke; Workpiece: Ck45N.

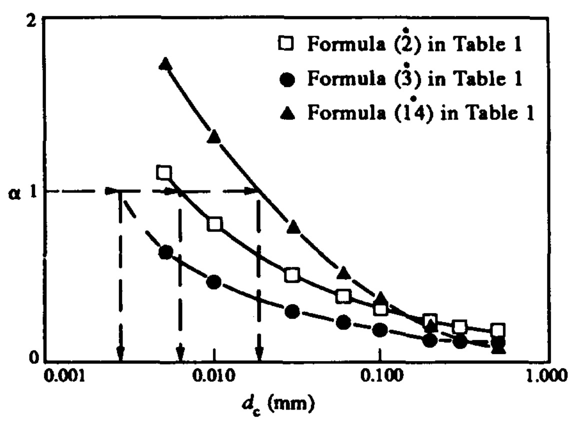

FIG. 2. Violation of validity criterion (equation (9)).

creep-feed grinding. Figures 4–7 show that this formula can fit all of these available results very well, and that its prediction accuracy is better than the others (see, e.g. Figs 5–7). We are therefore confident in saying that the new formula (7) is able to respond to most conditions in the ranges of conventional and creep-feed grinding. The form of this formula is also very simple. It only needs a few numbers of input data: F'_n, d_c, E_s, ν_s, R_0, d_n and ζ, where the condition coefficient, ζ, could be determined by only one set of measured results. For the same type of formulae, ($\dot{8}$) also needs one set of measured data, while ($\dot{4}$) needs two and ($\dot{7}$) needs three.

If we just look at Fig. 5, we find that formulae ($\dot{1}$), ($\dot{8}$) and ($\dot{9}$) produce results with almost the same order of accuracy, although ($\dot{1}$) has a little superiority among them. By considering the convenience of application, however, formula ($\dot{1}$) is the simplest. Formula ($\dot{9}$) involves the extra measurement of surface roughness, and formula ($\dot{8}$) uses the equivalent elastic modulus of a wheel–workpiece system that is not necessary because the effect of workpiece material has been reflected by F'_n and d_c.

4. CONCLUSIONS

(1) From the viewpoint of range of validity, accuracy of prediction and convenience of practical application, the formula proposed by the present paper

$$L_c = R_d \arccos\left(1 - \frac{d_c}{R_d}\right)$$

is better.

(2) It has been emphasized that any approximate model is applicable to a certain kind of grinding condition, if, and only if, this formula satisfies condition (9), i.e.

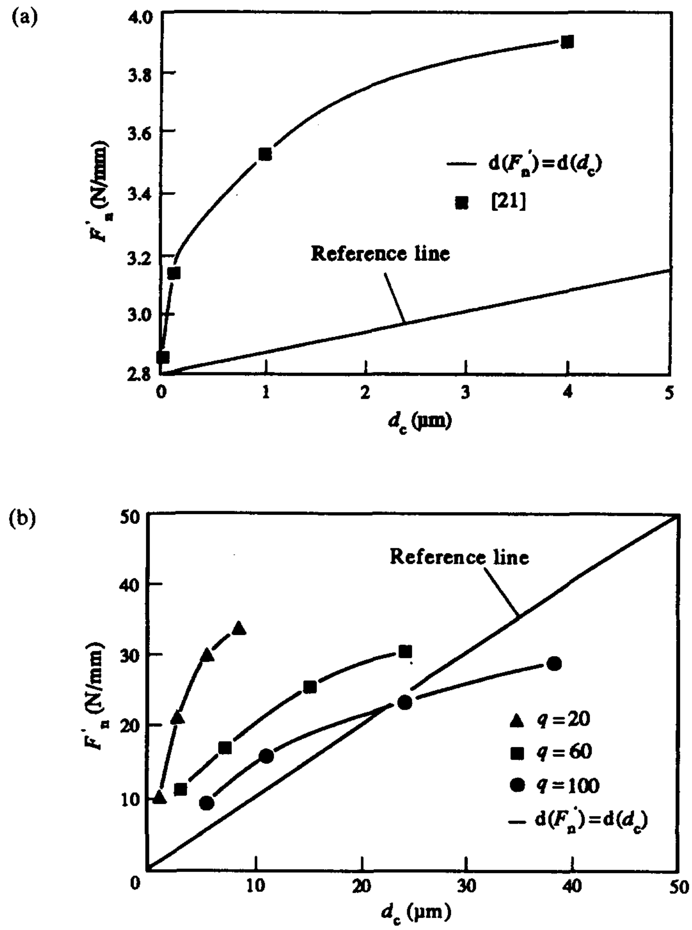

FIG. 3. Variation of F'_n with d_c: (a) Salje et al. measurements [21]; (b) Verkerk measurements [8].

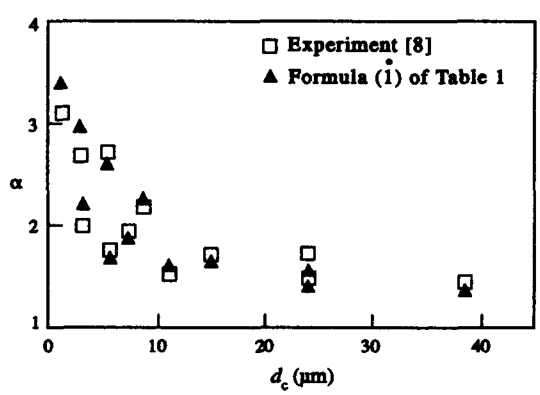

FIG. 4. A comparison of prediction of equation (7) with experimental data from Figs 4 and 5 of Ref. [8].

$$1 \leq \frac{L_c}{L_g} < N$$

in that particular range.

(3) Macro-deformation of a wheel–workpiece system is one of the most important factors that contribute to the variation of contact length during grinding.

(4) To make an approximate formula applicable to a wide range of grinding conditions, however, it is necessary to arrange an empirical parameter to compensate the

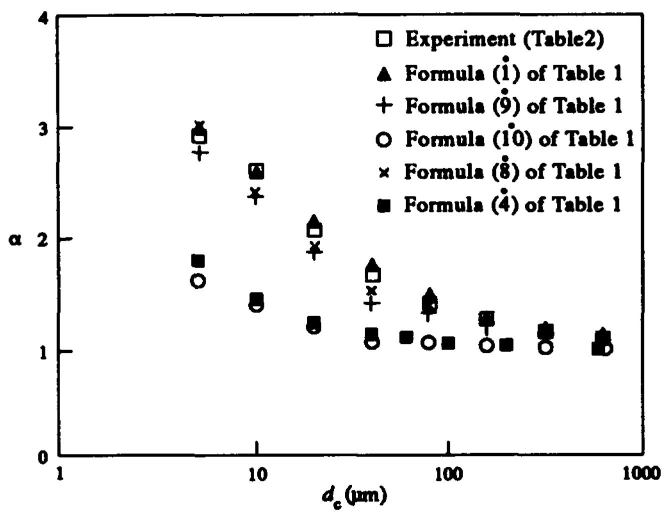

FIG. 5. A comparison of prediction of equation (7) with experimental data listed in Table 2.

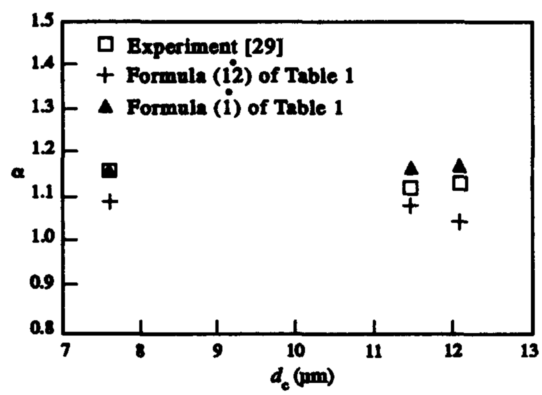

FIG. 6. A comparison of prediction of equation (7) with experimental data from Table 1 of Ref. [29].

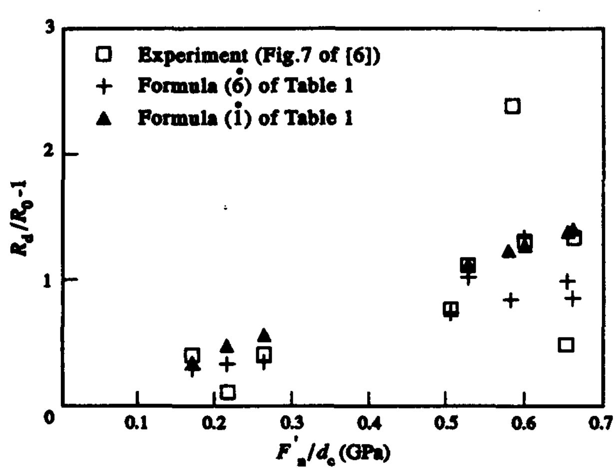

FIG. 7. A comparison of prediction of equation (7) with experimental data from Ref [6].

effects from other grinding variables that have not directly been considered in its formulation.

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