Grinding force modelling: combining dimensional analysis with response surface methodology

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Abstract: This paper describes a new technique for an empirical modelling of unit grinding force in surface grinding by combining dimensional analysis with Response Surface Methodology (RSM). The grinding experiment on K1045 steel (190 BHN) was carried out using conventional alumina wheels. The predictive grinding force model was developed in terms of spindle speed, work speed, depth of cut, width of cut and the strength of the work material. The adequacy of the model was judged by a variance analysis. Based on the response model, contours in the planes of investigated parameters were generated to predict the grinding conditions for a particular grinding force. The new technique can significantly reduce the number of experiments required for the force modelling and thereby is cost-effective.

Keywords: grinding; dimensional analysis; response surface methodology; design of experiment.

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1 Introduction

Grinding is a complex material removal operation involving many difficult-to-control parameters (Zhang et al., 1992), but is a major manufacturing process accounting for about 20–25% of the total expenditures on machining operations in industrialised countries (Malkin, 1989). Mathematical machinability models are often very useful for designing a suitable data selection in Computer Integrated Manufacturing System (CIMS) in industry. This paper aims to develop a mathematical model for unit grinding force in surface grinding by combining dimensional analysis with the Response Surface Methodology (RSM) to reduce the number of tests.

Dimensional analysis has been applied to the metal cutting (Kronenberg, 1966; Shaw and Oxford, 1957), grinding (Zhang et al., 1993, Lu et al., 1994), polishing (Sun et al., 2000) and drilling (Zhang et al., 2001) to develop empirical equations. They found that the relationships between key dimensionless parameters often follow a rule and that when such relationships had been established, they could be used to predict a wide range of applications. The behaviour of specific cutting energy in terms of the metal removal rate per unit width in grinding, explored by Malkin (1989), is a good example.

The conventional technique of one variable at a time to study the machining responses are time consuming and costly. In their investigation on the optimisation of chemical processes, Box and Wilson (1951) proposed the RSM to combine the design of experiments with the regression modelling technique for fitting a model to experimental data and statistical inferences. This method can substantially reduce the number of experiments required to study the relationship among independent variables and their responses in a system. Many investigators (Alauddin et al., 1996; Choudhury and El-Baradie, 1999) developed mathematical models for cutting forces in turning and milling by RSM. Naik et al. (1993) developed a surface roughness model for centreless grinding process. Kwak (2005) presented an application of RSM for modelling the geometric error in terms of four process parameters in surface grinding. He used orthogonal design of experiments having 27 experiments for developing the second-order model.

This study aims to develop an empirical mathematical model of unit grinding force in surface grinding in terms of five factors: spindle speed, work speed, depth of cut, width of cut and tensile strength of work material. Dimensional analysis will be used to work out a functional, non-homogeneous equation that links the individual variables/parameters by a small number of dimensionless groups. Then, by utilising the design of experiment of RSM, a mathematical model will be developed in terms of these dimensionless parameters, which can significantly reduce the number of grinding tests.

2 Dimensional analysis

The factors affecting the grinding force (Hou and Komanduri, 2003; Kalpakjian and Schmid, 2001; Kwak, 2005; Zhang et al., 1993) are shown in Figure 1. However, the grinding force for a given set of wheel-workpiece system can be considered to be the function of the following easily controllable factors:

$$F_g = f\left(V_s, V_w, d, w, S_u\right)$$

where F_g is grinding force, V_s spindle speed, V_w work speed, d depth of cut, w width of cut and S_u is tensile strength of the workpiece material. These variables are listed in Table 1 together with three primary units, that is, mass M, length L and time T.

Figure 1 Factors influencing the grinding forces in surface grinding

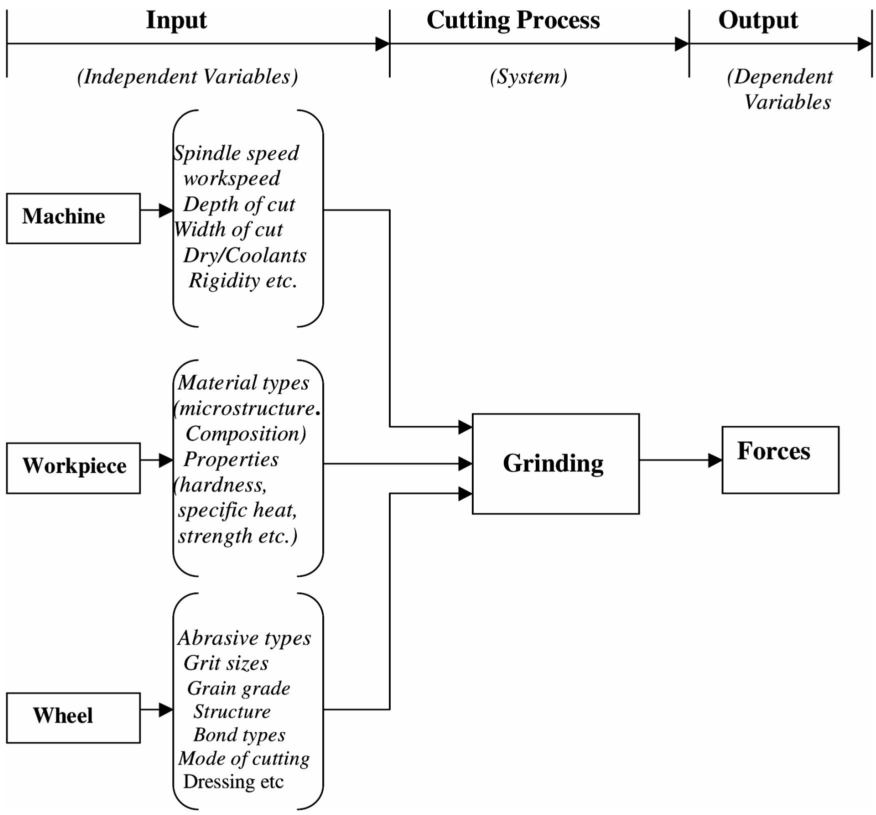


Table 1 List of variables and their dimensional formula

Variable number	*Variable name*	*Symbol of variable*	*Dimensional formula*
1	Grinding force	F_g	MLT^{-2}
2	Spindle speed	V_s	LT^{-1}
3	Work speed	V_w	LT^{-1}
4	Depth of cut	D	L
5	Width of cut	W	L
6	Tensile strength	S_u	$ML^{-1}T^{-2}$

The Buckingham's π-theorem of dimensional analysis indicates that the relationship among the variables in the functional form can be expressed as:

$$f_1\left(F_g, V_s, V_w, d, w, S_u\right) = 0$$

The above 6 variables contain 3 primary units, that is, M, L and T. According to the π-theorem, there exist 3 independent dimensionless products or π-terms. The construction of the 3 π-terms rests upon the fact that the best set should provide most insight to the problem under investigation (Sharp et al., 1992). Based on these, for the present surface grinding process, the above equation can be written as

$$f_2(\ \pi_1,\ \ \pi_2,\ \ \pi_3)\ \ =\ \ 0$$

or

$$f_2\left(\frac{V_w}{V_s}, \frac{d}{w}, \frac{F_g V_s}{V_w d w S_u}\right) = 0$$

or

$$\pi_3 = \frac{F_g V_s}{V_w d w S_u} \quad = f_3\left(\frac{V_w}{V_s}, \frac{d}{w}\right)$$

The π-theorem of dimensional analysis suggests that the functional relationship between the response/dependent parameter (π_3) and independent parameters (π_1 and π_2) can be written as:

$$\hat{\pi}_3 = C\left(\frac{V_w}{V_s}\right)^{\alpha}\left(\frac{d}{W}\right)^{\beta} \tag{1}$$

where $\hat{\pi}_3$ is the predicted response (dependent dimensionless parameter) and C, α, β are the model parameters to be determined by experiment.

Taking natural logarithm converts the intrinsically linear type non-linear model into standard linear form of the first-order regression model as:

$$\hat{y} = b_0 x_0 + b_1 x_1 + b_2 x_2 \tag{2}$$

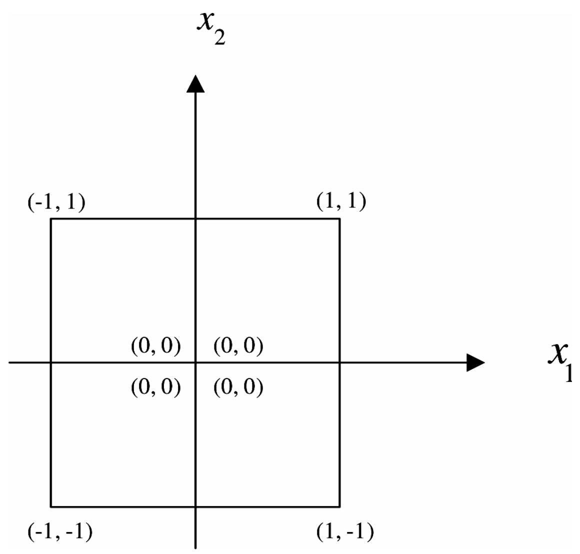
where $\hat{y}$ is the predicted response on natural logarithmic scale, $x_0 = 1$ (a dummy variable), x_1, x_2 are the coded values (logarithmic transformations) of V_w/V_s, d/w respectively, and b_i (i = 0, 1, 2) are parameters to be estimated by linear multiple regression.

3 Response surface methodology

3.1 Experimental design of fitting the model

A well-designed experiment can substantially reduce the number of experiments. RSM can be most efficient if proper attention is given to the choice of experimental design. The preferred class of response surface design is a factorial design in which all levels of a given factor are combined with all levels of every other factors in the experiment. This design is necessary when interactions between variables are important. Furthermore, a factorial design allows the effect of a factor to be estimated at several levels of the other factors, leading to conclusions applicable to a broader range of experimental conditions (Montgomery, 1981). An important design methodology is the 2^K factorial design which allows to study the joint effect of K factors, each at only two levels, on a response. These levels may be quantitative, such as two values of a factor or qualitative, such as the high and low levels of a factor. However, there is also a special case of 2^K factorial design with n centre points, or replicated runs, added to the centre of the design; that is, n replicated runs at medium levels of factors. Replicated runs at a centre point of the design allow one to check for quadratic effects (curvature) and to obtain an independent error estimation. The inclusion of centre points to the 2^K design does not affect the regression coefficeints (b_i), but the estimation of b_0 becomes the grand average of all observations. The factorial design of experiment with 2 factors is shown schematically in Figure 2, consisting of four corner points of a square and a central point repeated four times to estimate the pure error.

Figure 2 Factorial design for 2 factors



3.2 Estimation of test regions (coding) for independent variables

To simplify the calculation, the independent variables are coded so that their values are found in the interval (−1, 0, 1). The levels of independent variables and coding identifications used in the design of experiment are presented in Table 2. The coded values of the variables for use in Equation (2) are obtained from the following transforming Equations (Lo and Chen, 1977):

$$x_1 = \frac{\ln(V_w / V_s) - \ln(7.5/(25\times 60))}{\ln(10/(25\times 60)) - \ln(7.5/(25\times 60))} \tag{3}$$

$$x_2 = \frac{\ln(d/w) - \ln(10/25\times 10^3)}{\ln(15/25\times 10^3) - \ln(10/25\times 10^3)} \tag{4}$$

where x_1 is the coded value of the factor corresponding to its natural value V_w/V_s, (7.5/(25 × 60)) natural value of this factor corresponding to the base or 0 level value, (10/(25 × 60)) the natural value of this factor at the +1 level and x_2 is the coded value of the factor corresponding to its natural value d/w, (10/(25 × 10^3)) the natural value of this factor corresponding to the 0 level, (15/(25 × 60)) the coded value of this factor at +1 level.

Table 2 Coding and levels of Factors

Factors	*Units*	*Symbol*	*Coding*	*Levels of factors*		
				Low (−1)	*Medium (0)*	*High (+1)*
1. Work speed/ Spindle speed	$\frac{\text{m/sec}}{\text{m/sec}}$	V_w/V_s	x_1	5.6/25 × 60	7.5/25 × 60	10/25 × 60
2. Depth of cut/ Width of cut	mm/mm	d/w	x_2	6.5/25 × 10^3	10/25 × 10^3	15/20 × 10^3

3.3 Experiment

The process utilised for measuring grinding forces was a surface up-grinding operation, performed on a CNC Surface grinder (Junior 90 CNC-M286, Fratelli Minini SNC). The work-piece material was a steel, 190 BHN (fully killed carbon steel, K1045, S_u, tensile strength = 630 MPa). The grinding tests were carried out using grinding wheels based on aluminium oxide (Al_2O_3) abrasives. The specification of the grinding wheel is WA60HV and its dimensional size (diameter/width) is 300 mm/25 mm. The grinding tests were carried out under coolant (water and oil mixture = 30:1) conditions. The average tangential grinding force (F_g) was measured by a Kistler piezo-electric dynamometer. The results of eight tests and their analysis are listed in Tables 3 and 4 together with the actual grinding conditions and coding identification.

Table 3 Experimental conditions and results

Expt. Run	*Grinding conditions*				*Factors*		*Level of factors (coding)*		*Grinding force F_g (N)*	*Dependent parameter π_3 (F_gV_s/V_wdwS_u)*
	Spindle speed V_s (m/sec)	*Work speed V_w (m/min)*	*Depth of cut d (μm)*	*Width of cut w(mm)*	*V_w/V_s (π_1)*	*D/W (π_2)*	x_1	x_2		
1	25	5.6	6.5	25	0.0037	0.00026	–1	–1	28	73.26
2	25	10	6.5	25	0.0067	0.00026	1	–1	31	45.42
3	25	5.6	15	25	0.0037	0.0006	–1	1	68	77.10
4	25	10	15	25	0.0067	0.0006	1	1	78	49.52
5	25	7.5	10	25	0.005	0.0004	0	0	58	73.65
6	25	7.5	10	25	0.005	0.0004	0	0	51	64.76
7	25	7.5	10	25	0.005	0.0004	0	0	53	67.30
8	25	7.5	10	25	0.005	0.0004	0	0	46	58.41

Table 4 Analysis of results

Expt. run	*π_3 (observed) (F_gV_s/V_wdwS_u)*	*$\hat{\pi}_3$ (predicted)*	*y (= Inπ_3) (observed)*	*$\hat{y}$ = (ln $\hat{\pi}_3$) (predicted)*	*Unit grinding force k_s (Observed) J/sec*	*Unit grinding Force $\hat{k}_s$ (Predicted) J/sec*
1	73.26	76.23	4.2940	4.3338	46.15	47.89
2	45.42	48.11	3.8160	3.8734	28.61	30.35
3	77.10	81.66	4.3451	4.4026	48.57	51.41
4	49.52	51.72	3.9025	3.9422	31.20	32.59
5	73.65	62.66	4.2993	4.1377	46.40	39.48
6	64.76	62.66	4.1707	4.1377	40.80	39.48
7	67.30	62.66	4.2092	4.1377	42.40	39.48
8	58.41	62.66	4.0675	4.1377	36.80	39.48

3.4 *Estimation of parameters*

On the basis of the data given in Table 3 the values of regression coefficients (b_0, b_1, b_2) in the regression model were estimated by the least square method, using the following basic formula written in a matrix form (Drapper and Smith, 1966).

$$b_{\mathrm{m}} = (X_{\mathrm{m}}^{\mathrm{T}} X_{\mathrm{m}})^{-1} X_{\mathrm{m}}^{\mathrm{T}} Y_{\mathrm{m}} \tag{5}$$

where b_{m} is the matrix of parameters, X_{m} the design matrix of levels of independent variables x, X^{T} the transpose of matrix X_{m} and Y_{m} is the matrix of logarithm of the measured response y.

The X_{m} and Y_{m} matrices can be written in the following form:

$$X_m = \begin{bmatrix} 1 & -1 & -1 \\ 1 & +1 & -1 \\ 1 & -1 & +1 \\ 1 & +1 & +1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad Y_m = \begin{bmatrix} 4.2940 \\ 3.8160 \\ 4.3451 \\ 3.9025 \\ 4.2993 \\ 4.1707 \\ 4.2092 \\ 4.0675 \end{bmatrix}$$

The vector of the estimated regression coefficients is found to be

$$b_m = \begin{bmatrix} 4.1380 \\ -0.2302 \\ +0.0344 \end{bmatrix}$$

3.5 Analysis of the results

3.5.1 Development of the mathematical model

The predictive model can be obtained by substituting the estimated parameters into Equation (2), which gives rise to

$$\hat{y} = 4.1380 - 0.2302\, x_1 + 0.0344\, x_2 \tag{6}$$

This equation can be transformed using Equations (3) and (4) to provide the dependent dimensional parameter as the function of the two independent dimensionless parameter, that is,

$$\hat{\pi}_3 = \frac{F_c V_s}{V_w d w S_u} = 1.8868 \left(\frac{V_w}{V_s}\right)^{-0.7865} \left(\frac{d}{w}\right)^{0.0849} \tag{7}$$

From Equation (7) the predicted unit grinding force (work done per unit volume of material removed) $\hat{k}_s$ can be shown to be

$$\frac{\hat{F}_c V_s}{V_w d w} = \hat{k}_s = 1.8868\, S_u \left(\frac{V_w}{V_s}\right)^{-0.7865} \left(\frac{d}{W}\right)^{0.0849} \tag{8}$$

Equation (8) indicates that an increase in the ratio of the feed rate to wheel speed decreases the unit grinding force while an increase in the ratio of depth of cut to width of cut increases the unit grinding force. This equation is valid for surface up-grinding of K1045 steel (190 BHN) using a grinding wheel of the specification of WA60 HV with coolant and the following range of grinding conditions:

Spindle speed: 25 m/sec

Feed rate: 5.6–10 m/min

Depth of cut: 6.5–15 μm

Width of cut: 25 mm

The predictive Equation (6) can be plotted as contours for each of the responses as shown in Figure 3 for determining the optimum grinding condition for a required grinding force. From the contours shown in the Figure 3, it is possible to select a combination of grinding conditions (i.e. speed, feed, depth of cut and width of cut) that produce the same machining response. Figure 4 shows the 3D response surface of dependent dimensionless machining parameter against the two independent dimensionless parameters. The differences between measured and predicted unit grinding force are illustrated in Figures 5 and 6.

3.5.2 Adequacy of the predictive model

The variance (Analysis of Variance (ANOVA)) technique can be used to check the adequacy of the predictive model because it can provide information of the *p*-value and the R^2 value (Montgomery et al., 2001). The *p*-value is the probability that the test statistic will take on a value. It is the smallest level (α) of significance at which the model or data are significant. Once the *p*-value is known, the decision maker can draw a conclusion at any specified level of significance. Usually, *p*-value smaller than 0.05 signifies that the particular terms of a model have significant effect on the model. The coefficient of multiple correlations, R^2 is a measure of the fraction of total variation in the data accounted by the model. Because of the analysis of variance identity, its value must be $0 \le R^2 \le 1$. The model fitting assessment is given in Table 5.

The results of *p*-value and R^2 statistics above show that the predictive model is adequate.

Figure 3 Machining response contours for π_3 parameter in the π_1–π_2 parameter plane (log-scale)

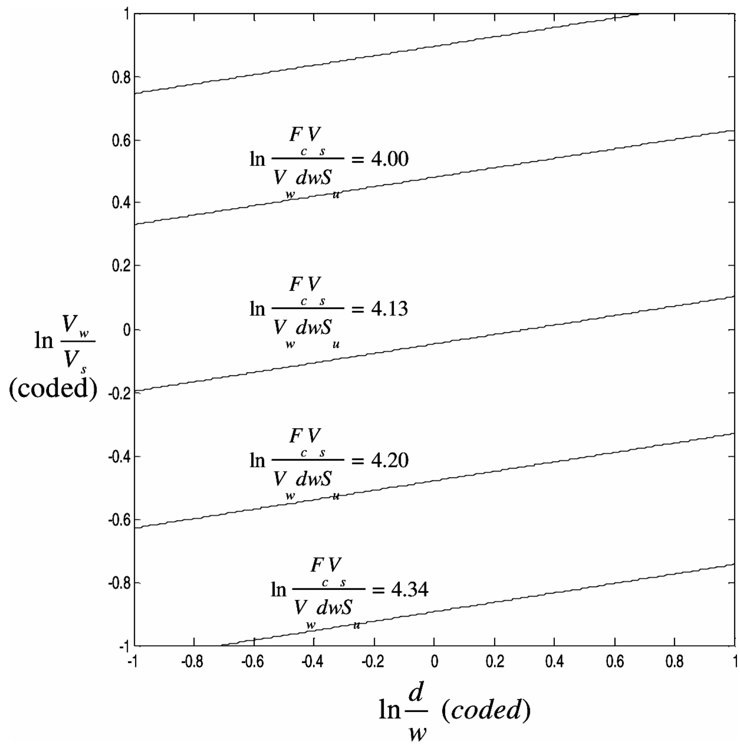


Figure 4 3D plot of the response surface for π_3 parameter in the π_1–π_2 parameter mesh

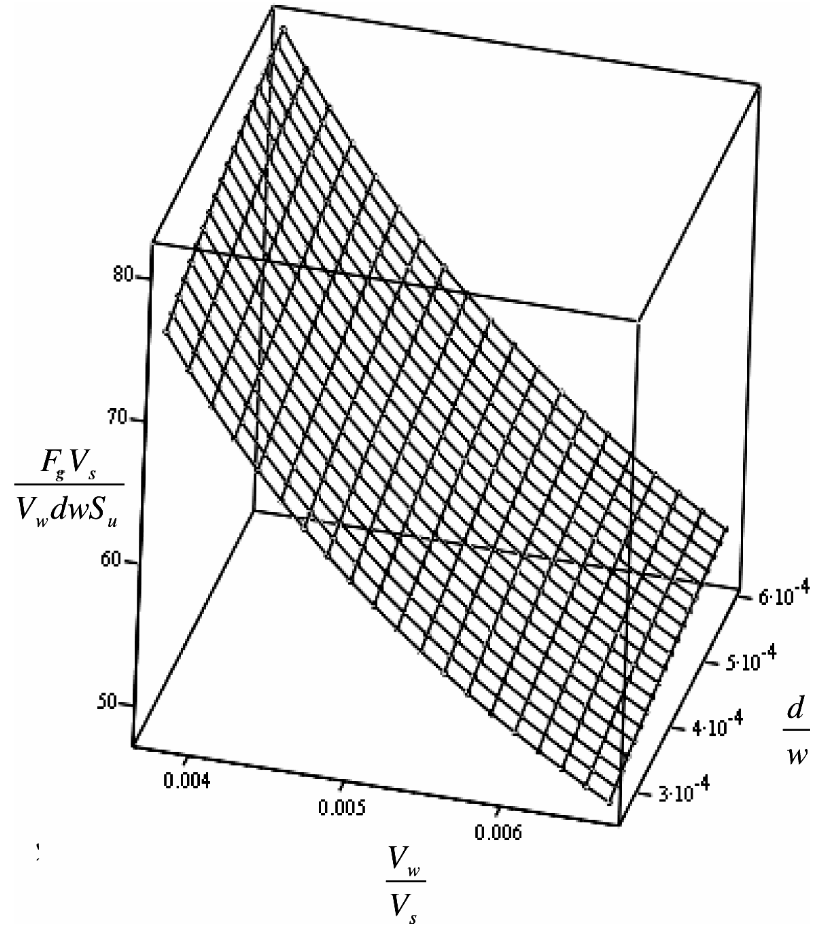


Figure 5 Measured versus predicted unit grinding force

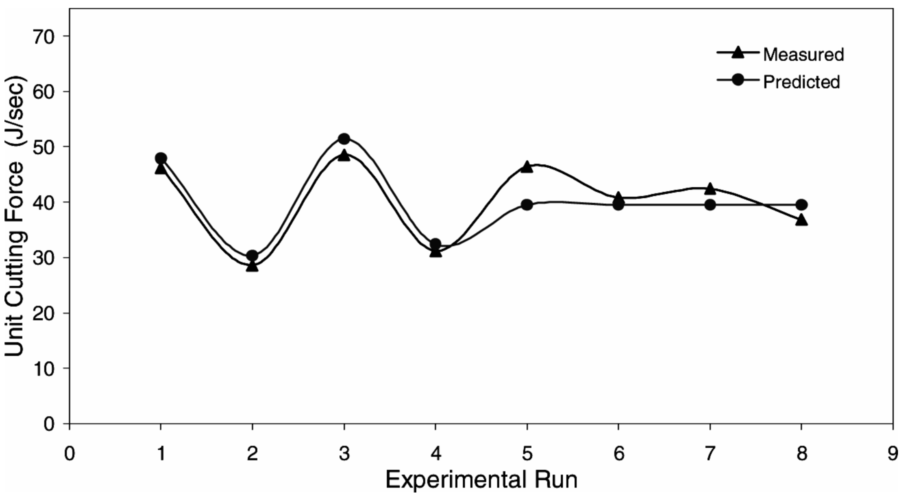


Figure 6 Relative prediction variation of unit grinding force

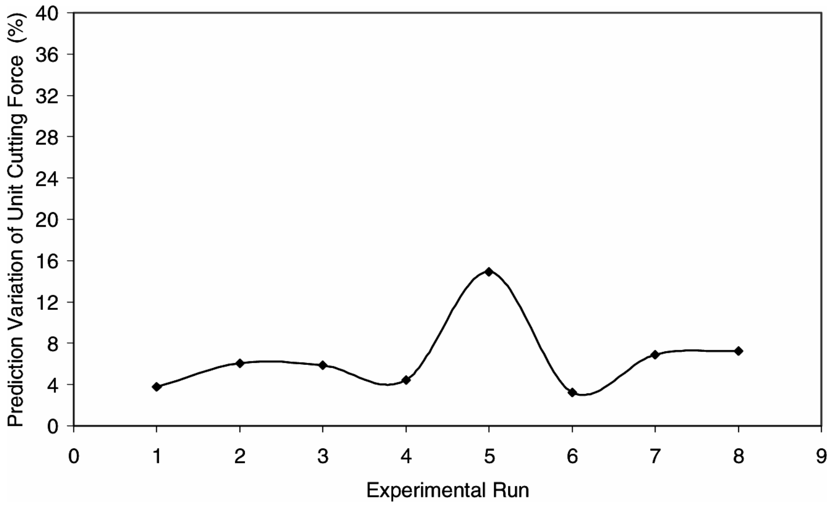


Table 5 ANOVA for the predictive model of dependent dimensionless parameter

Source	*Sum of squares*	*Degrees of freedom*	*Mean square*	*p-Value*	R^2
Model	0.216609591	2	0.108304795	0.0134*	0.8221
Residuals	0.046887832	5	0.009216		
Total	0.263497422	7	0.037642489		

*0.0134 > 0.05.

4 Conclusions

i A mathematical model for predicting the unit grinding force has been developed for surface grinding, which combines the dimensional analysis with the RSM to significantly reduce the number of grinding tests.

ii The model predicts that the unit grinding force decreases with the increase of work to wheel speed ratio and increases with an increase of either the strength of the material or the depth to width of cut ratio.

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